

INTERGENERATIONAL TRANSFERS AND THE FERTILITY-INCOME RELATIONSHIP*

INTERGENERATIONAL TRANSFERS AND FERTILITY

Juan Carlos Córdoba and Marla Ripoll

Abstract

Evidence from cross-sectional data reveals a negative relationship between family income and fertility. This paper argues that constraints to intergenerational transfers are crucial for understanding this relationship. If parents could legally impose debt obligations on their children to recover the costs incurred in raising them, then fertility would be independent of parental income. A relationship between fertility and income arises when parents are unable to leave debts because of legal, enforcement, or moral constraints. This relationship is negative when the intergenerational elasticity of substitution is larger than one, case in which parental consumption is a good substitute for children's consumption.

There is extensive empirical evidence documenting a negative relationship between fertility and income.¹ For example, using cross-sectional individual data for the US, Becker (1960) finds a negative fertility-income relationship in the 1910, 1940 and 1950 Censuses, and in the Indianapolis survey for the 1900s. More recently, Jones and Tertilt (2008) use US Census data as far back as the 1826 cohort to estimate an income elasticity of fertility of about -0.38 . Their analysis is distinct in that they construct a more refined measure of lifetime income by using occupational income and education. Lifetime income and fertility are measured for several cross-sections of five-year birth cohorts from 1826-1830 to 1956-1960. They conclude that most of the observed fertility decline in the US can be explained by the negative fertility-income relationship estimated for each cross-section, together with the outward shift of the income distribution over time. The estimated income elasticity is robust to the inclusion of additional controls such as child mortality and the education

*Corresponding author: Marla Ripoll, Department of Economics, University of Pittsburgh, 4532 W.W. Posvar Hall, Pittsburgh PA 15260, United States, Email: ripoll@pitt.edu.

An earlier version of this paper circulated under the title: "Barro-Becker with Credit Frictions." We are grateful for the comments we received from the editor, two anonymous referees, Daniele Coen-Pirani, Xiyang Liu and participants at seminars at the University of North Carolina Chapel Hill and Penn State University, as well as the following conferences: the "Progressive Economic Research Conference" at the Richmond Fed, the 2010 and 2011 Midwest Macroeconomics Conferences, the 2011 North American Meeting of the Econometric Society, the Economic Growth Workshop at the 2011 NBER Summer Institute, and the 2011 "Toward Sustainable Economic Growth" Workshop sponsored by IAE, CREI and Barcelona GSE.

¹See Jones and Tertilt (2008) and Jones *et al.* (2011) for a recent survey of the literature.

of husband and wife, suggesting a strong negative correlation between income and fertility. What explains the observed negative fertility-income relationship?

This paper shows that constraints to intergenerational transfers are central to understanding the negative fertility-income relationship. We show that if parents could legally impose debt obligations on their children as a way to recover the costs incurred in raising them, then fertility would be independent of parental income. In particular, if the present value of a child's future income exceeds the cost of raising the child, parents would have incentives to raise as many children as possible in order to maximise rents. Altruistic parents would have extra incentives because, on top of the financial benefit, they would also enjoy their children. Absent constraints to intergenerational transfers, fertility would not be associated to parental income. In contrast, if parents cannot impose debt obligations on their children either because of legal, moral or enforcement reasons, then the relationship between fertility and parental income can be rationalized. Specifically, we show that debt limits, or bequest constraints, bind precisely when children are a net financial gain to society, i.e., the cost of raising the child is below the present value of the child's future income.

Available data discussed in Section 2 indicates that in effect, children are a net financial gain in the sense of Becker and Barro (1988). Consider for instance a child from a low income family. According to the USDA (2012), the typical cost of raising a child born in 2011 from age 0 to 17 for a family of two adults and two children in the low income group is \$169,080. Using a discount rate of 1.5% this corresponds to a present value of \$148,962. This figure includes direct parental expenses made on children through age 17 such as housing, food, transportation, health care, clothing, child care, and private expenses in education, but abstracts from time costs. Accounting for the time costs of raising children is not trivial, but the best available estimates suggest a range of between \$223,443 and \$446,886 for a low income child. Therefore, the total cost of a low income child stands at around \$372,405 to \$595,848 in present value. To obtain an estimate of the present value of this child's income, the life-cycle profile fitted in Kambourov and Manovskii (2009) can be used under the assumptions that the child works from age 18 to 65, and that the parent and the child have the same income at age 28 (allowing for a real income growth of 1% per year). As discussed in Section 2, this procedure yields a present value of the child's future income of \$661,529. These calculations suggest that a low income child generates more lifetime income than what it costs to raise him. Net financial gains for children in middle and high income families are even higher, as we discuss

in Section 2. Although these children cost more, they also generate more income on average.

The nature of the constraints to intergenerational transfers is to a large extent legal. Schoonbroodt and Tertilt (2010) provide a summary of how the legal control parents have over their children's lifetime labour income has changed over time in the US and England. While prior to 1850 parents in these countries had legal access to their offspring's labour income, starting in the mid-19th century many reforms were passed that removed the legal ability of parents to own the earnings of their children. For instance, starting in the early 19th century, parents in the US lost their right to legally use their children as servants in exchange for the expenses incurred in raising them. It became a legal obligation to parents to provide appropriate care of their children. In addition, by 1938 child labour had been banned, and compulsory schooling had been enforced all over the US. In sum, at least since the early 19th century, parents cannot legally impose debt obligations on their children to recover the costs of raising them.² From the perspective of modeling fertility choices, this historical evolution suggests that the negative fertility-income relationship documented above for the US plausibly requires the explicit consideration of limits to intergenerational transfers.

In addition to showing how constraints to intergenerational transfers are essential in understanding the relationship between fertility and income, our paper contributes to the literature on fertility choice by providing conditions under which this relationship is negative in models of dynastic altruism. Since both the marginal cost and the marginal benefit of children depend on wages, a negative fertility-wage relation can only be obtained when marginal costs increase proportionally more than marginal benefits. We show that there are at least two channels by which this may occur. The first one is when the elasticity of utility with respect to consumption increases with consumption. Intuitively, an increasing elasticity makes parental consumption a superior good reducing the option value of having a child. For the case of CRRA utility, an increasing elasticity is only possible if the *elasticity of intergenerational substitution*, the one controlling the willingness to substitute consumption between parents and their children, is larger than one. The reason is that a parent who can easily substitute own and children's consumption has a lower marginal gain of adding a child. In other words, adding one more child requires spreading consumption among more family members, which is more costly for a parent who cares little about consumption smoothing across generations.

²See Section 2 in Schoonbroodt and Tertilt (2010).

A second instance arises when there exists an exogenous income component, for example, government transfers. This exogenous component increases consumption and therefore the incentives to have children, but it affects low-wage individuals more than high-wage individuals generating a negative fertility-income relationship. Since only one of the two channels just discussed is necessary, it is feasible to obtain plausible conditions under which the relationship between fertility and income is negative.

Beyond offering an explanation for the negative fertility-income relationship in cross-sectional data within countries, our model also helps explaining cross-country and time-series data. The poorest countries in the world still exhibit very high levels of fertility, levels that are similar to those experienced by today's developed nations at the start of their demographic transitions. Dynastic altruistic models can account for this evidence in the presence of constraints to intergenerational transfers, an intergenerational elasticity of substitution larger than one, and either an exogenous income component (non-labour income) or non-homothetic preferences. Our analysis also suggests that the absence of a legal environment that effectively prevents parents from extracting rents from their children may be one of the reasons why fertility levels are so high in a number of countries.³ Rent extraction may induce maximum fertility regardless of whether parental income is high or low.

Our paper also contributes to the literature by integrating dynastic and overlapping generation models. Since constraints to intergenerational transfers are binding in our model, savings and aggregate capital are mainly determined by life-cycle considerations, but families still manage to smooth welfare across generations by adjusting fertility. Issues of Ricardian equivalence and the role of social security in achieving efficiency are usually studied in the context of exogenous fertility. For instance, Weil (1987) and Abel (1987) show that Ricardian equivalence does not hold when bequest constraints are binding, and that social security transfers are not fully compensated by bequests.⁴ This paper complements this literature by deriving conditions under which bequest constraints bind in an altruistic OLG model with endogenous fertility.⁵ We also derive conditions under which a negative fertility-income relationship is obtained both in partial and general equilibrium.

³Doepke (2004) develops a theory of demographic transition based on the related idea that changes to child labor regulation laws can help explain the demographic transition. Our theory does not rely on changes to child labor regulation laws.

⁴We thank an anonymous referee for pointing this out.

⁵See Lapan and Enders (1990) for a related paper on this point.

Although the literature on endogenous fertility choices is vast, only a handful of papers have explicitly modeled how constraints to intergenerational transfers affect fertility choices. Many models of dynastic altruism do not consider constraints to intergenerational transfers, while others shut down asset markets without discussing the underlying rationale of this choice.⁶ A notable recent exception is Schoonbroodt and Tertilt (2014), who examine efficiency issues in an altruistic model of endogenous fertility and constraints to parental ownership of children's income. Our paper complements theirs, and it is the first paper to explicitly rationalize the role of constraints to intergenerational transfers in accounting for the observed negative fertility-income relationship.

There is also a related literature in which parents are not purely altruistic, but they care about the number, and sometimes about the quality, of their children.⁷ Although this strand of the literature has made important contributions to the understanding of a number of empirical regularities, most notably the demographic transition, these models implicitly assume zero bequests, and therefore do not discuss the role of constraints to intergenerational transfers.⁸ Interestingly, our paper shows that when bequest constraints bind in dynastic altruistic models a link can be established between purely altruistic and some non-altruistic models. This link suggests that both approaches may result in similar predictions when constraints to intergenerational transfers are properly considered. Schoonbroodt and Tertilt (2014) arrive to a similar conclusion in their analysis of efficiency.

The remainder of the paper is organized as follows. Section 1 develops a dynastic altruistic model of endogenous fertility, derives the main results of the paper regarding the fertility-income relationship, and obtains the conditions under which bequest constraints bind in partial equilibrium. Data on the cost of raising children and the present value of children's future income is presented in Section 2. Section 3 extends the results to general equilibrium. Concluding comments are provided in Section 4 and technical details are presented in the appendix.

⁶Dynastic altruistic models in the tradition of Becker and Barro (1988) and Barro and Becker (1989), along with most of the subsequent work surveyed in Jones *et al.* (2011) abstract from intergenerational transfer constraints. A few papers, including Becker *et al.* (1990), Doepke (2004), De la Croix and Vander Donckt (2010), and Jones and Schoonbroodt (2014), shut down all asset markets. We thank an anonymous referee for pointing this out.

⁷Notable examples in this category of papers include Galor and Weil (2000), Jones (2001), Greenwood and Seshadri (2002), Kalemli-Ozcan (2003), Tamura (2006), Galor *et al.* (2009), and Ashraf and Galor (2011).

⁸For a more extensive discussion on this see Schoonbroodt and Tertilt (2014).

1 A constrained model of dynastic altruism

This section considers fertility decisions by altruistic parents in a partial equilibrium OLG economy, or alternatively, in a small open economy. General equilibrium considerations are postponed to Section 3. The main purpose of this section is twofold. First, it derives and interprets the conditions under which constraints to intergenerational transfers bind, and it shows how in this case fertility and parental income are related. Second, it derives and discusses the conditions under which the fertility-income relationship is negative.

1.1 Altruistic preferences

Consider a model in which time is discrete and individuals live for three periods: one as children, one as working young adults, and one as retirees. A young adult or parent at time t consumes c_{1t} in period t and c_{2t+1} in period $t + 1$, has n_t children at time t , and derives utility

$$V_t = U(c_{1t}, c_{2t+1}) + \Phi(n_t)V_{t+1} \text{ for } t = 0, 1, \dots, \quad (1)$$

where $U(c_{1t}, c_{2t+1}) \geq 0$ is a utility flow, $\Phi(n_t) \geq 0$ is the weight parents place on the welfare of their n children, $n \in [0, N]$, and $V_{t+1} \geq 0$ is the utility of a child. Assume $U(c_{1t}, c_{2t+1}) = u(c_{1t}) + \rho u(c_{2t+1})$, where u is a non-negative utility function and $\rho > 0$. Further assume $\Phi(0) = 0$, $\Phi'(n) > 0$, $\Phi''(n) < 0$ and $\Phi(N) < 1$. The last three restrictions imply that parental altruism is positive, decreasing in the number of children, and bounded.⁹ In particular, the average degree of altruism, $\beta(n) \equiv \Phi(n)/n$, decreases with n .

The (marginal) willingness to pay for a child, WTP , is a key determinant of the demand for children. It is measured by the marginal rate of substitution between parental consumption and children as defined by

$$\overline{WTP}(n_t, c_{1t}, V_{t+1}) \equiv \frac{\partial c_{1t}}{\partial n_t} = \frac{\partial V_t / \partial n_t}{\partial V_t / \partial c_{1t}} = \Phi'(n_t) \frac{V_{t+1}}{u'(c_{1t})}. \quad (2)$$

⁹As in Becker and Barro (1988), we only consider the positive utility case. Alvarez (1999), Barro and Sala-i-Martin (2004) and Jones and Schoonbroodt (2009, 2010) analyze cases with negative utility by allowing $\Phi'(n) < 0$. We do not consider this possibility because it is intrinsically inconsistent with fully altruistic parents. Jones and Schoonbroodt (2009) show that there is an interpretation of their problem under which "parents are only weakly altruistic toward their children" (p. 3). Cordoba and Ripoll (2011, example 2) formalize the idea of "altruism" and show that the extension with $\Phi'(n) < 0$ violates the fundamental axiom of altruism. Moreover, a framework with positive utility can handle any elasticity of intertemporal substitution.

The first component of this expression, $\Phi'(n_t)$, is the altruistic weight parents give to the n -th child, while the second component is the welfare of the child measured in (parental) consumption units. $\overline{WTP}(n, c, V)$ decreases with n and increases with c_1 and V . Below, we focus on a steady state situation in which allocations and welfare are constant over time. In this case equations (1) and (2) simplify to

$$V = \frac{(1 + \rho) u(c)}{1 - \Phi(n)}, \text{ and} \quad (3)$$

$$WTP(n, c) = (1 + \rho) \frac{u(c)}{u'(c)} \frac{\Phi'(n)}{1 - \Phi(n)} = \frac{1 + \rho}{\epsilon^u(c)} \frac{\Phi'(n)}{1 - \Phi(n)} c, \quad (4)$$

where $\epsilon^u(c) = u'(c)(c/u(c)) > 0$ is the elasticity of the utility function with respect to consumption. When the elasticity is a constant independent of c , the marginal valuation of a child is proportional to consumption. This means that the steady state expansion path on the consumption-children space is linear, so that doubling income would double the valuation of a child. This channel alone works toward producing a positive relationship between fertility and income. However, the effect is weakened when the elasticity $\epsilon^u(c)$ is an increasing function of consumption. In that case parental consumption is like a superior good and as a result richer parents are less willing to trade their own consumption for children. As we formalize below, once budget and intergenerational transfer constraints are introduced, an increasing elasticity is the first channel that could explain the negative fertility income relationship.

Examples of utility functions that exhibit an increasing elasticity are (i) CRRA functions of the form $u(c) = c^{1-\sigma}/(1-\sigma) + A$ with $\sigma \in (0, 1)$ and $A > 0$; and (ii) Stone-Geary functions of the form $u(c) = (c - a)^{1-\sigma}/(1-\sigma)$ with $\sigma \in (0, 1)$ and $a < 0$. Both of these cases require relative mild curvature of the utility function, $\sigma \in (0, 1)$. Examples of utility functions that exhibit decreasing elasticity are the CRRA case with $\sigma > 1$, Stone-Geary with a minimum consumption requirement, $a > 0$, and CARA functions, $u(c) = 1 - e^{-\theta c}$. We discuss the CRRA case in detail below. The high curvature case is problematic when rationalizing the negative fertility-income relationship because it implies strong diminishing marginal utility of parental income, which would cause richer parents to put a relatively higher value on children and thus demand more children.

1.2 Constraints: parental budget and intergenerational transfers

Young adults work, raise children and save. Retirees use savings to consume and bequeath to their children. Bequests are restricted to be non-negative. Raising a child involves three costs: a time cost of δw , a goods cost of κ , and bequests b_{t+1} per child. While δ and κ are technological parameters, b_{t+1} is a choice variable.¹⁰ The restriction $\delta N \leq 1$ is required so that the total time invested in children does not exceed available parental time. The parental budget constraint is

$$w + y + b_t = c_{1t} + \frac{c_{2t+1}}{1+r} + n_t \left(\delta w + \kappa + \frac{b_{t+1}}{1+r} \right),$$

where w is the wage rate, r the interest rate, and y is an exogenous income component or non-labour income (e.g., government transfers).

In addition to the budget constraint, individuals face constraints to intergenerational transfers in the form of non-negative bequest constraints: $b_{t+1} \geq 0$ for all $t \geq 0$. As discussed in the introduction, these restrictions may reflect legal or other constraints making it unfeasible to enforce debt contracts on descendants. Some parameter restrictions are needed for bequest constraints to bind. We postulate the key assumption in this section and show its relevance below when characterizing optimal choices.

ASSUMPTION 1. BINDING BEQUEST CONSTRAINTS. (i) $\lim_{n \rightarrow 0} (1+r)\beta(n) < 1$ for all n and/or (ii) $w + y > (1+r)(\delta w + \kappa)$.

The first case in which bequest constraints bind, condition (i), is one in which the interest rate is low and/or parents do not attach enough weight to their children. As in models with impatient agents, it is optimal in this situation for early members of the dynasty to borrow and finance a relatively high consumption while later members pay debts and consume less. Similar conditions are derived in Weil (1987) and Abel (1987) for bequest constraints to bind in an OLG model with exogenous fertility. Assumption 1 (i) is satisfied by functions $\Phi(n) = \alpha(1 - e^{-\mu n})$ or $\Phi(n) = (\alpha\mu n)/(1 + \mu n)$ under the restriction $(1+r)\mu < 1$.

A second case arises when the cost of raising children is lower than future children's earnings, as described in Assumption 1 (ii). This assumption describes the case in which parents could financially benefit from having children. Absent bequest constraints, parents could borrow to pay

¹⁰As in Becker and Barro (1988), the technology of raising a child is Leontief in parental time and goods.

all costs of raising children and then let their children pay these debts by endowing each child with negative bequests. Parents could also extract rents by leaving negative bequests that are larger than the cost of raising the child, but not so large as to eliminate any consumption by the child. Binding constraints to intergenerational transfers prevent such solutions to arise in equilibrium. Assumption 1 (ii) is of particularly empirical relevance. Section 2 provides detailed evidence on the costs of raising children and estimates of children's future income. As we show there, the evidence strongly suggests that children are a net financial benefit in the sense that the cost of raising a child is lower than the present value of the child's earnings.

The following assumption is made for convenience and it is relaxed in Section 3. It generates a simple flat life-cycle consumption profile but none of the main results depend on this particular assumption.

ASSUMPTION 2. INTEREST RATE IN PARTIAL EQUILIBRIUM. $\rho(1+r) = 1$.

1.3 *Individual's problem and decision rules*

It is convenient to formulate the individual's problem as a dynamic programming problem.¹¹ Let $V(b)$ be the maximum lifetime utility of a parent who inherits b . The young adult's problem is to choose consumptions, c_1 and c_2 , fertility n , and bequest, b' , that solve the problem

$$V(b) = \max_{c_1, c_2, b' \geq 0, N \geq n \geq 0} U(c_1, c_2) + \Phi(n)V(b'), \quad (5)$$

subject to

$$w + y + b \geq c_1 + \frac{c_2}{1+r} + n \left(\delta w + \kappa + \frac{b'}{1+r} \right). \quad (6)$$

The first order condition for optimal savings and bequests can be expressed respectively as

$$u'(c_1) = (1+r)\rho u'(c_2), \text{ and} \quad (7)$$

$$u'(c_1) \geq (1+r)\beta(n)u'(c'_1), \text{ with equality if } b' > 0. \quad (8)$$

¹¹See Appendix A.1 for technical details about the proper boundness and transversality conditions of the endogenous fertility problem.

The first equation is the traditional Euler equation describing optimal intra-generational consumption smoothing. Assumption 2 together with equation (7) imply $c_1 = c_2$.¹² The second condition is an intergenerational Euler equation describing optimal consumption smoothing across parents and their children. The inequality reflects the possibility of binding bequest constraints. Savings, on the other hand, are unconstrained. The relevant discount factor controlling optimal bequests is endogenous and corresponds to the average degree of altruism toward children, $\beta(n)$. When bequest constraints are not binding, (8) describes a quantity-growth trade-off: consumption growth is negatively related to fertility. As shown below, when constraints are binding the trade-off is between fertility and the consumption level rather than consumption growth.

The optimality condition for fertility is given by¹³

$$u'(c_1) \left(\frac{b'}{1+r} + \delta w + \kappa \right) = \Phi'(n)V(b'). \quad (9)$$

The left hand side of equation (9) is the marginal cost of a child. It includes the cost of raising a child plus the present value of any bequests, both multiplied by the marginal utility of consumption. The marginal benefit, on the other hand, is the welfare of the child, V , times the parental weight associated to the last child, $\Phi'(n)$.

As pointed out by Alvarez (1999), fertility decisions are analogous to investment decisions. To see this, rewrite (9) as

$$u'(c_1) = (1+r^n) \Phi'(n)u'(c'_1), \quad (10)$$

where

$$1+r^n \equiv \frac{V(b')/u'(c'_1)}{b'/(1+r) + \delta w + \kappa},$$

is the gross return of having a child. This is the case because V/u' is the value of a life in terms of goods while $b'/(1+r) + \delta w + \kappa$ is the marginal cost of creating a life. Comparing (10) and (8) shows that fertility decisions are analogous to investment decisions. Moreover, returns on children

¹²Identical solutions for fertility can be obtained if instead of Assumption 2, $u(c)$ is assumed to be CRRA. In this case, equation (7) can be written as: $c_2 = \gamma(r, \rho)c_1$ where $\gamma(r, \rho) = u'^{-1} \left(\frac{1}{(1+r)\rho} \right)$.

¹³For simplicity, we discuss only cases where optimal fertility is an interior solution. Appendix A.2 describes the assumptions on the functional forms of $u(c)$ and $\Phi(n)$ required for the existence of an interior solution for fertility.

and bequests must satisfy the arbitrage condition

$$(1 + r^n) \Phi'(n) \geq (1 + r) \Phi(n)/n.$$

Since $\Phi(n)$ is concave, this condition implies that in an interior solution $1 + r^n > 1 + r$, meaning that returns to fertility must be larger than returns to financial assets. This fertility premium is required because additional savings allow parents to provide each child with larger bequests increasing parental utility in proportion to the average degree of altruism, $\Phi(n)/n$, but having an extra child, holding bequests constant, increases the utility of the parent only in proportion to the marginal degree of altruism, $\Phi'(n)$.

1.4 *Constrained allocation: steady state*

We now characterize the steady state of the model and focus on cases in which Assumption 1 holds. We first show that bequest constraints are binding under Assumption 1 (i). In steady state the optimality condition for bequests in equation (8) simplifies to

$$1 \geq (1 + r) \beta(n), \text{ with equality if } b' > 0. \quad (11)$$

Since $\beta(n)$ decreases with n , Assumption 1 (i) implies $(1 + r) \beta(n) < 1$ for all n and therefore $b = 0$ is the only possible solution to (11).

Let n^* and $c^* = c_1^* = c_2^*$ be the steady state solutions when bequest constraints bind. In this case (5) simplifies to (3) while (6) and (9) simplify to

$$c^* = \frac{w + y - n^* (\delta w + \kappa)}{1 + \rho}, \text{ and} \quad (12)$$

$$\delta w + \kappa = (1 + \rho) \frac{\Phi'(n^*)}{1 - \Phi(n^*)} \frac{u(c^*)}{u'(c^*)} = WTP(c^*, n^*). \quad (13)$$

The last expression equates the marginal rate of transformation to the marginal rate of substitution between parental consumption and children. The left hand side of (13) is the marginal cost of raising a child while the right hand side is the marginal benefit, what was defined above as the willingness to pay for a child. According to (13) and (12), optimal fertility, n^* , generally depends on w and y in the constrained case. As we show below, this stands in sharp contrast with the determinants

of fertility in the unconstrained allocation, where fertility is not determined from such comparison of costs and benefits of children. It is interesting to notice the similarity between equation (13) and the determinants of fertility in some non-altruistic models. For instance, in Greenwood and Seshadri (2002, p. 156) fertility is determined by equalizing marginal costs and marginal benefits of children. Marginal costs include the opportunity wage cost of children, and the benefits are a function of the future wage of the child.

Equations (12) and (13) can be used to obtain a graphical solution of c^* and n^* . Equation (12) provides a negative relationship between consumption and fertility, while (13) provides a positive relationship. This last relationship follows because $u(c)/u'(c)$ is an increasing function of c .¹⁴

The following proposition characterizes the effects of non-labour income and wages on consumption and fertility.

PROPOSITION 1. COMPARATIVE STATICS. *Let Assumptions 1 and 2 hold. Then $\partial c^*/\partial y > 0$, $\partial n^*/\partial y > 0$ and $\partial c^*/\partial w > 0$ while $\partial n^*/\partial w$ is undetermined.*

PROOF. *See Appendix A.3.*

According to Proposition 1, both consumption and fertility increase with non-labour income, y . Furthermore, while the effect of wages on fertility is undetermined, higher wages increase consumption. The model thus provides plausible comparative statistics for consumption.

1.5 *The negative fertility-income relationship*

According to equations (12) and (13), wages affect optimal fertility decisions in three ways. First, higher wages increase the time cost of raising children which alone would lead to a lower fertility by high wage parents. Second, higher wages also increase consumption and welfare of children, and in fact of all descendants via the term $u(c)$, an effect that tends to generate a positive fertility-wage relationship. Finally, higher wages and higher parental consumption reduce the marginal utility of consumption, $u'(c)$, making the utility gains from having children more valuable. As explained in Section 1.1, the stronger the diminishing marginal utility of income the higher the willingness to pay for a child. Whether $\partial n^*/\partial w$ is positive or negative depends on which effect dominates. One

¹⁴ Assumption A.1 in Appendix A.2 guarantees the existence and uniqueness of a single crossing.

can rewrite equation (13) as follows so that only two effects need to be considered

$$\frac{\delta w + \kappa}{w + y - n^* (\delta w + \kappa)} = \frac{1}{\epsilon^u(c^*)} \frac{\Phi'(n^*)}{1 - \Phi(n^*)}. \quad (14)$$

The left hand side of equation (14) is the marginal cost of a child relative to consumption while the right hand side is the marginal benefit also relative to consumption. Given that the relative marginal benefit is decreasing in n^* (Assumption A.1), fertility decreases with wages when the relative marginal cost increases with wages or the relative marginal benefit decreases with wages.

Equation (14) allows to identify two fundamental reasons why fertility may decrease with wages. The first one is when the relative marginal cost increases with wages. A simple derivative shows that this occurs when $y > \kappa/\delta$. Thus, a positive and sufficiently large amount of non-labour income y , or a sufficiently low goods-intensity of raising a children κ/δ , lead to a higher relative cost of raising children and a lower demand for children. To understand why, consider for a moment the case $y = 0$ and $\kappa/\delta > 0$. In that case the relative marginal cost strictly decreases with wages because higher wages proportionally increase income w , while it increases costs $\delta w + \kappa$ less than proportionally. On the other hand, if $y > 0$ but $\kappa = 0$ then higher wages increase costs δw proportionally, while income $w + y$ increases less than proportionally. In the borderline case $\kappa = \delta y > 0$, the relative marginal cost becomes independent of wages.

The second reason is when the relative marginal benefit decreases with wages. This occurs when the elasticity of utility with respect to consumption, $\epsilon^u(c^*)$, is an increasing function of consumption since, as stated in Proposition 1, consumption is an increasing function of wages. As discussed in Section 1.1, in that case parental consumption is like a superior good and as a result richer parents are increasingly less willing to trade their own consumption for a child. The following proposition summarizes these results.

PROPOSITION 2. FERTILITY-INCOME RELATIONSHIP. *Fertility falls with wages if either of the following inequalities hold, and at least one of them is strict:*

- (i) *non-labour income and cost of raising children are such that $y > \kappa/\delta$;*
- (ii) *preferences are such that $\partial \epsilon^u(c)/\partial c \geq 0$;*

PROOF. *Follows directly from equation (14): condition (i) guarantees that the left-hand-side is increasing in w , while condition (ii) implies that the right-hand-side is decreasing with w .*

We now discuss the second channel in Proposition 2 in more detail. In order to more broadly

explore the implications of this preferences channel, consider CRRA utility $u(c) = c^{1-\sigma}/(1-\sigma) + A$, where $1/\sigma$ is the intertemporal elasticity of substitution. When $\sigma > 1$ constant $A > 0$ is needed for utilities to remain positive.¹⁵ For this utility function $\epsilon^u(c) = (1-\sigma)/[1 + (1-\sigma)Ac^{\sigma-1}]$. Notice that when $\sigma > 1$, $\epsilon^u(c)$ decreases with consumption and furthermore $\lim_{c \rightarrow \infty} \epsilon^u(c) = 0$. This reveals a key feature of fertility choices with CRRA utilities: individuals choose maximum fertility when the elasticity of substitution is below one and wages are sufficiently high. In terms of equation (13), the marginal value of income, $u'(c) = c^{-\sigma}$, falls at a faster rate than the rate at which the opportunity cost of raising children increases as $w \rightarrow \infty$. As a result, the relative marginal cost of raising children decreases toward zero while the relative marginal benefit remains bounded above zero making maximum fertility optimal. In contrast, when the elasticity of substitution is above one, fertility can be an interior solution. In this case, $\sigma \in (0, 1)$, $\epsilon^u(0) = 0$ and $\lim_{c \rightarrow \infty} \epsilon^u(c) = 1-\sigma$. Thus, the relative marginal cost of children for poor individuals is zero and therefore they choose the maximum number of children. These results are formalized by the following proposition.

PROPOSITION 3. CRRA UTILITY AND FERTILITY WHEN $w \rightarrow \infty$ AND $w \rightarrow 0$. *Let $u(c) = \frac{c^{1-\sigma}}{1-\sigma} + A \geq 0$ where $\sigma \geq 0$ and $\sigma \neq 1$. Then, $\lim_{w \rightarrow \infty} n^*(w) = N$ if $\sigma > 1$ and $\lim_{w \rightarrow \infty} n^*(w) = \min[\tilde{n}^*, N]$ if $\sigma < 1$ where \tilde{n}^* solves the equation*

$$\frac{\delta}{1 - \delta\tilde{n}^*} = \frac{1}{1 - \sigma} \frac{\Phi'(\tilde{n}^*)}{1 - \Phi(\tilde{n}^*)}. \quad (15)$$

Furthermore, $\lim_{w \rightarrow 0} n^*(w) = N$ if $\sigma \in (0, 1)$.

PROOF. See Appendix A.4.

The result with $\sigma > 1$ resembles one obtained by Hall and Jones (2007) for the case of longevity. They show that longevity is a superior good when $\sigma > 1$ because the marginal utility of consumption falls with income, which implies that the marginal cost of longevity also falls, while the marginal benefit of longevity does not. Although such feature of preferences is convenient to explain the increasing demand for longevity, it renders the case $\sigma > 1$ inconsistent with the negative fertility-income relationship.

When $\sigma \in (0, 1)$, fertility approaches a value determined by (15). For the case $\Phi(n) = \alpha(1 - e^{-\mu n})$, equation (15) becomes $(1 - \sigma)\delta[(1 - \alpha)e^{\mu\tilde{n}} + \alpha] = \alpha\mu(1 - \delta\tilde{n})$, which has a unique

¹⁵Furthermore, consumption has to be above $[(\sigma - 1)A]^{\frac{1}{1-\sigma}}$ to guarantee $u(c) \geq 0$. When $\sigma > 1$, A corresponds to the maximum utility flow while the minimum utility flow is zero.

solution if $(1 - \sigma) \delta < \alpha \mu$. Thus, fertility for high wage individuals ($w \rightarrow \infty$) depends negatively on the time cost of raising children, δ , and positively on the degree of altruism, α .

Proposition 3 creates a tension between the theory and the empirics: while an intertemporal elasticity ($1/\sigma$) lower than one is typically used in applied work, this elasticity would have the counterfactual implication that the most productive individuals would have the highest fertility.¹⁶ Intuitively, a low elasticity of substitution increases the option value of providing positive consumption to a newborn as wages increase. In contrast, if the elasticity is larger than one, then high parental consumption can substitute for low or zero consumption of descendants. But a closer look at this intuition reveals that the key parameter is not the intertemporal elasticity of substitution, which controls intra-personal consumption smoothing, but the *intergenerational elasticity of substitution* which controls inter-personal consumption smoothing. When preferences are additive separable, as we assume, both elasticities are equal to $1/\sigma$. In a companion paper, Cordoba and Ripoll (2011), we show that if these two elasticities are disentangled using non-separable models, then the tension described above is resolved: the most productive individuals do not need to have the highest fertility even if the intertemporal elasticity of substitution is below one as long as the intergenerational elasticity is above one. The practical implication of these results is that for fertility issues it is better to interpret $1/\sigma$ in separable models as the intergenerational rather than the intertemporal elasticity of substitution.

While Proposition 3 refers only to high or low wage individuals, the following Lemma provides a necessary and sufficient condition for fertility to decrease with wages for all wage levels for the case of CRRA preferences.

LEMMA 1. CRRA UTILITY AND FERTILITY. *Let $u(c) = \frac{c^{1-\sigma}}{1-\sigma} + A \geq 0$ where $\sigma \geq 0$. Then $\partial n^*/\partial w < 0$ if and only if*

$$1 + \frac{y}{w} > [\Psi(c^*) (1 - \delta n^*) + \delta n^*] \left(1 + \frac{\kappa}{\delta w}\right), \quad (16)$$

where

$$\Psi(c^*) = \sigma + \epsilon^u(c) = \sigma + \frac{1 - \sigma}{1 + (1 - \sigma) A c^{\sigma-1}}.$$

PROOF. *See Appendix A.5.*

¹⁶See discussion in Jones *et al.* (2011).

The following proposition uses Lemma 1 to typify two polar cases in which fertility decreases with wages. While $\sigma \in (0, 1)$ is needed in both cases, the first case also requires strictly positive non-labour income while the second requires non-homothetic preferences.

PROPOSITION 4. NON-LABOUR INCOME AND NON-HOMOTHETIC UTILITY. *Let $u(c) = \frac{c^{1-\sigma}}{1-\sigma} + A \geq 0$ where $\sigma \geq 0$. Then if $0 < \sigma < 1$ fertility decreases with wages in the following cases:*

- (i) *non-labour income case: $A = 0$ and $y > \kappa/\delta$;*
- (ii) *non-homothetic preferences: $\kappa = y = 0$ and $A > 0$.*

PROOF. *Follows from condition (16) in Lemma 4. Notice that in case (ii) $\Psi(c^*) < \Psi(\infty) = 1$.*

The first part of Proposition 4 requires a sufficiently large non-labour income component. If $y > 0$ and labour is the only input in the production of children, then the condition is satisfied for all wages, and $\partial n^*/\partial w \rightarrow 0$ as $w \rightarrow \infty$. The second part of Proposition 4 assumes $y = 0$ but sets $A > 0$. This non-homotheticity of the utility function generates a decreasing relative marginal benefit of children as w increases.

Proposition 4 highlights the importance of either non-labour income or non-homothetic utility in order to generate a negative fertility-income relationship under the constrained allocation. Although this is indeed the case here, we have verified in ongoing work that the presence of non-labour income or non-homothetic utility are not required in generalizations of the constrained model to settings with uncertainty.¹⁷ In those settings, what is essential to obtain the negative fertility-income relationship is the non-negative bequest constraint.

In conclusion, once bequest constraints bind, there are instances in which a negative fertility-income relationship holds, as described in Propositions 2 and 4. The conditions described in these propositions are not strong, as only one of them should hold with strict inequality for the results to go through. Are these conditions novel? Are they plausible? Some of these conditions, specifically the presence of non-labour income and non-homothetic utility, have been discussed in the fertility literature, but in the context of static models of fertility choice.¹⁸ One of the novelties of Propositions 2 and 4 is that they are derived in the context of a dynamic model of fertility in which the constraints to intergenerational transfers are explicitly considered. As we show next subsection, in the absence of these constraints, fertility is independent of income.

The requirement that the intergenerational elasticity of substitution is larger than one, i.e.,

¹⁷See Cordoba *et al.* (2014) and Cordoba and Liu (2014).

¹⁸See Jones *et al.* (2011).

the case $\sigma < 1$ discussed above, is novel. This condition makes intuitive sense if we associate intergenerational substitution as the relevant long-run concept, while the more known intertemporal substitution is thought of as relevant for the short run. There is evidence in the macroeconomics literature that individuals have low willingness to substitute consumption across periods, so the smoothing motive is strong in the short run. The fertility evidence on the other hand suggests high willingness to substitute consumption across generations. The notion of intergenerational substitution as the relevant long-run concept is also novel.

1.6 *Unconstrained intergenerational transfers*

In contrast with the constrained case discussed so far, many of the altruistic models of fertility in the tradition of Becker and Barro (1988) assume unconstrained bequests. We now characterize the unconstrained solution and compare it with the constrained solution. The most important message of this subsection is that in the absence of constraints to intergenerational transfers, fertility is independent of income. This is the case both for the steady state and for the transition of the model.

Let n and $c = c_1 = c_2$ be the steady state solutions of the unconstrained problem. In this case the intergenerational Euler equation (8) holds with equality, and in steady state it reads

$$1 = (1 + r) \beta(n). \tag{17}$$

According to this expression, steady state fertility is a sole and positive function of the interest rate but it is independent of level variables such as w or y . The unconstrained model thus does not generate the sort of negative fertility-income relationship observed in the data. The fundamental reason for this prediction is that steady state fertility is fully determined by the intergenerational Euler equation, an equation in which level variables do not play a role, rather than by the first order condition for fertility, a condition that in contrast includes level variables. While equation (8) would determine the balanced growth rate of individual consumption in a standard model with exogenous fertility, in models with endogenous fertility it determines the discount factor, which is endogenous and a function of steady state fertility.¹⁹

¹⁹The lack of a relationship between fertility and income implied by equation (17) does not mean that children are not normal goods. It reflects instead that all fertility adjustments to income changes take place during the transition. In the Barro-Becker model, for example, the effects of all present and future income changes on fertility take place

Once fertility is determined by the interest rate, steady state consumption is determined by

$$c = \frac{1}{1 + \rho} \frac{\epsilon^u(c)}{\epsilon^\Phi(n) - \epsilon^u(c)} [(1 + r)(\delta w + \kappa) - w - y],$$

where $\epsilon^\Phi(n) = \Phi'(n)(n/\Phi(n)) > 0$ is the elasticity of the altruistic function with respect to the number of children. According to this expression, consumption is positive only if children are a net financial burden to parents and to society: $(1 + r)(\delta w + \kappa)$ needs to be larger than $w + y$. This prediction is at odds with the data presented in the introduction and expanded in Section 2, which suggests children are not a net financial burden. Notice that if Assumption 1 (ii) holds, then the unconstrained solution is not possible because consumption would be negative. Assumption 1 (ii) thus provides a sufficient, but not necessary condition, for bequest constraints to bind. The condition is sufficient because it holds for all n . A necessary and sufficient condition can be derived as follows. Given fertility and consumption, optimal bequests are determined from the budget constraint as

$$b = \frac{(1 + \rho)c - (w + y) + n(\delta w + \kappa)}{(1 + r - n)/(1 + r)}.$$

Equation (17) implies that $1 + r - n = (1 + r)(1 - \Phi(n)) > 0$. Therefore, steady state bequests are non-negative if $(1 + \rho)c + n(\delta w + \kappa) > w + y$. The following is a necessary and sufficient condition for $b < 0$.

PROPOSITION 5. BINDING BEQUEST CONSTRAINT. *Let n solve $1 = (1 + r)\beta(n)$. Then a necessary and sufficient condition for bequest constraints to bind is*

$$\frac{\epsilon^u(c)}{\epsilon^\Phi(n)} \frac{n(\delta w + \kappa)}{w + y - n(\delta w + \kappa)} < \frac{n}{1 + r - n},$$

where $\epsilon^\Phi(n)$ is the elasticity of function Φ with respect to n .

PROOF. *This proposition follows after algebraically transforming condition $(1 + \rho)c + n(\delta w + \kappa) < w + y$, which as shown above induces negative bequests in the unconstrained case.*

In order to interpret Proposition 5, assume that $\kappa/w \approx 0$ which describes a rich individual in which w is high relative to κ , and suppose $y = 0$. In this case, the condition in Proposition 5 only in the first period since their economy reaches a steady state in the second period. Without bequest constraints, the present value of all incomes is accrued by the first generation, and for isoelastic preferences, it is optimal for only the first generation to increase its fertility. Intuitively, more income means more branches out of the trunk of the tree, but consumption and fertility remain the same on each branch. For non-isoelastic cases the situation is different, but it becomes the same as the isoelastic case in the steady state.

becomes

$$\frac{\epsilon^u}{\epsilon^\Phi} \frac{\delta n}{1 - \delta n} < \frac{n}{1 + r - n} = \frac{\Phi(n)}{1 - \Phi(n)},$$

where the last equality uses equation (17). Since $\epsilon^u/\epsilon^\Phi < 1$, this condition states that the weight that parents give to their children, $\Phi(n)$, needs to be larger than the total fraction of time spent raising children, δn . According to this condition, bequest constraints bind if ϵ^u/ϵ^Φ , the interest rate, and the cost of raising children are sufficiently low, or if wages are sufficiently high.²⁰ In sum, the unconstrained model, or the Becker and Barro (1988) model, predicts fertility is independent of income and that children are a financial burden to the parents.

These predictions hold not only for the steady state, but also along the transition. To see this, consider equation (8) which still holds with equality in the unconstrained model. Assuming CRRA utility, equation (8) becomes

$$\beta(n_t) = \left(\frac{c_{t+1}}{c_t} \right)^\sigma (1 + r_{t+1})^{-1},$$

where $1/\sigma$ is the intertemporal elasticity of substitution. This equation states that fertility is a positive function of the interest rate and a negative function of the growth rate of consumption. In other words, conditional on economic growth and interest rates, fertility is independent of the level of income. The evidence reported in the introduction suggests otherwise. For instance, Jones and Tertilt (2008) estimate an income elasticity of fertility of about -0.38 . Similar relationship between fertility and family income is reported by Lam (1986) for Brazil, and Kaur (2000) for India. The model with unconstrained intergenerational transfers cannot account for this evidence.²¹

2 Children as a net financial benefit

Assumption 1 (ii) provides a direct way to empirically explore a condition under which bequest constraints bind, namely that the cost of raising children is lower than the value of children's future earnings. When this is the case children represent a net financial benefit, a term coined in Becker and Barro (1988). Although the question of whether children are a net financial benefit is hard to

²⁰ A sufficient condition for this to hold is $(1 + r)\delta < 1$. In this case, using (17) it follows that

$$\frac{\Phi(n)}{1 - \Phi(n)} = \frac{1}{1/\Phi(n) - 1} = \frac{1}{(1 + r)/n - 1} > \frac{1}{1/(\delta n) - 1} > \frac{\epsilon_c^U}{\epsilon_n^\Phi} \frac{\delta n}{1 - \delta n}.$$

²¹The argument is analogous to why frictionless models cannot explain the documented relationship between individual schooling and individual income. See Cordoba and Ripoll (2013) for details.

answer, available evidence suggests this is the case, as we now discuss. According to the USDA (2012), the typical cost of raising a child born in 2011 from age 0 to 17 for a family of four in the lowest income group is \$169,080, while for a family in the middle-income group is \$234,900 and for a high-income family is \$389,670 in 2011 dollars. These are projected costs that assume an inflation rate of 2.55%. Assuming a discount rate of 1.5%, the corresponding present values of these sums are \$148,962 for low income, \$206,709 for middle income, and \$342,759 for the high income group. As discussed in the introduction these totals include direct parental expenses made on children through age 17 such as housing, food, transportation, health care, clothing, child care, and private expenses in education. These figures correspond to the goods costs of raising children.

Accounting for the time costs of raising children is not trivial. Available estimates are based on time use survey data, but the difficulty of measuring time costs is that in many instances parents multitask, taking care of children as a secondary activity while performing other primary activities. Using the 2003-2006 American Time Use Survey, Guryan *et al.* (2008) find that while mothers spend around 14 hours per week in child care, fathers spend around 7 hours. These measures only include the time parents spend primarily on basic care of children, education, recreation and any travel related to these. They refer to overall averages for families with at least one child under the age of 18.²² However, if the total time parents spend in the presence of their children is measured (both primary and secondary time), then mothers spend 45 hours per week and fathers spend 30 hours. The extent to which both primary and secondary time should be included in the cost of raising children is a matter of debate in the literature.

In a related study, Folbre (2008) uses the 1997 Child Development Supplement of the Panel Survey of Income Dynamics to conclude that the average amount of both passive and active parental-care hours per child (not including sleep) is 41.3 per week for a two-parent household with two children ages 0 to 11. Passive care corresponds to the time the child is awake but not engaged in activity with an adult, while active parental care measures the time the child is engaged in activity with at least a parent. In addition to reporting hours spent in child care, Folbre (2008) discusses two alternative ways of computing the monetary value of these hours: one uses a child-care worker's wage and the other the median wage. When the former method is used in combination with the USDA (2012) goods cost of raising children, the time cost of raising children is on average around

²²These estimates go up to 21 hours per week for mothers and around 10 hours for fathers in families with at least one child under the age of 5.

60% of the total costs (see Table 7.3, p. 135), a lower-end estimated. Since the median wage is around the double of a child-care worker's wage, then using the former time valuation the time cost of raising children increases to 75% of the total costs. This evidence suggests the time costs of raising children are high: they are between 1.5 and 3 times the goods costs or direct expenditures in children.

Combining the information above we compute bounds on the time cost of raising children for each of the USDA (2012) income categories. The results are presented in Table 1. Scenario 1 corresponds to the case in which time costs are 60% of the total costs (goods plus time) of raising children. This scenario holds either when the 21 hours per week of primary care in Guryan *et al.* (2008) are valued at the median wage, or when the 41.3 hours per week in Folbre (2008) are valued at a child-care worker's wage. Scenario 2 corresponds to the case in which time costs are 75% of the total costs of raising children. This scenario corresponds to the 41 hours per week of passive and active parental child care valued at the median wage as in Folbre (2008). As can be seen in Table 1, the time costs of raising children are between \$223,443 and \$446,886 for low-income families; between \$310,063 and \$620,127 for middle-income families; and between \$514,138 and \$1,028,227 for high-income families. All these figures correspond to present values at birth of the child in 2011 using a discount rate of 1.5%. Table 1 also presents the following bounds for the total costs of raising children: for a low-income family between \$372,405 and \$595,848; for a middle-income family the bounds are \$516,772 and \$826,836; and for a high-income family \$856,897 and \$1,371,036. Children in the United States are expensive to raise.

What is the value of children's future earnings? Table 2 presents estimates of the present value of children's future earnings for low, middle and high-income families. For this purpose, we first create a "representative" family for each income group. Using the family income brackets from USDA (2012), we select a 2011 income of \$36,665 for the representative low-income family; \$81,140 for middle income, and \$126,435 for a high-income family. The low-income family number is computed as the average of the 2011 earnings of a worker making the federal minimum wage, which is about \$13,920, and the upper bound of low-income families from the USDA (2012), which is \$59,410. The middle-income family number is simply the mid-point of the USDA (2012) interval of \$59,410 to \$102,280. Last, the high-income "representative" family is computed as the average between \$102,280 and \$150,000, which represents the 90th percentile of the family income distribution in

2011 according to the US Census Bureau.

Next, we compute the future earnings of a child from each income group as follows. Abstracting momentarily from college attendance, we assume children work from age 18 to 65. We also assume that the parent and the child have the same income at age 28, but taking into account that real income grows at a rate of 1% per year.²³ We use age 28 as a benchmark because it corresponds to the mean childbearing age in the US according to the OECD. To compute the whole earning profile from ages 18 to 65 we introduce life-cycle considerations. For this purpose we use the income polynomial estimated for the US in Kambourov and Manovskii (2009), in which log wages are expressed as a third-degree polynomial in age.²⁴ Using this procedure we obtain reasonable life-cycle income profiles with income peaking around age 41. Assuming a rate of discount of 1.5%, the present value of the child's future income is \$661,529 for the child from the low-income family, \$1,463,970 for the child from the middle-income family, and \$2,281,206 for the high-income family child. As shown in Table 2, these figures support the notion that children are a net financial benefit for parents in all income groups, since the child's income is above the total cost of raising him under either of the two scenarios of Table 1.

Two remarks on our computations are in order. First, the USDA (2012) direct expenditures on children include only ages 0 to 17, so college expenses are not included. Including college costs should not revert the net financial benefit on Table 2 to a net financial cost because it would most likely be children in middle and high-income families the ones attending college and the computed net benefits are sizeable. Second, our model predicts that when bequest constraints bind, no bequests or other inter-vivos transfers to adult children are given. Although (non-schooling related) inter-vivos transfers and voluntary bequests do occur in the United States, a relatively small fraction of adults receive them, and for the majority of them the amounts are small. For instance, using the 1988 special supplement on transfers between relatives from the PSID, Altonji *et al.* (1997) document that only 23% of adult children (on average 31 years old) receive transfers from parents (on average 59 years old). These are overall small transfers: the mean is \$1,810 in 1988 dollars, or \$3,442 in 2011 dollars; and the median is \$500 in 1988 dollars, or \$951 in 2011 dollars. A similar pattern has been documented for bequests. Using the 1993-1995 Asset and Health Dynamics

²³To compare the income of the child and the parent at age 28, we divide family income by two since the families in the USDA (2012) sample we use are two-parent families.

²⁴See their Table 5. According to the estimation, the coefficient on age is 0.0979, on age squared is -0.0029 , and that on age cubed is 0.00001.

among the Oldest Old (AHEAD) data, Hurd and Smith (2001) document that most bequests are of little or no value: single descendants at the bottom 30% receive \$2,000 (or \$2,952 in 2011 dollars), and the average single descendant receives \$10,000 (\$14,760 in 2011 dollars). Given the highly skewed wealth distribution in the United States, the occurrence of significant bequests concerns only of a small fraction of the population. The evidence on inter-vivos transfers and bequests does not overturn the facts document above regarding how for almost all parents in the United States, children are a net financial benefit.

3 General equilibrium

Consider now the joint determination of fertility, consumption, savings, wages and interest rates in a closed economy. In this section we show that the main results of Section 1 hold in general equilibrium. First, we show that in general equilibrium the negative fertility-wage relationship reflects a negative relationship between fertility and total factor productivity (TFP). Second, we discuss conditions for bequest constraints to bind in general equilibrium. We now explicitly introduce time subscripts, drop Assumption 2 and consider a specific functional form for u and a Cobb-Douglas production function. Following Proposition 4 (ii), assume $u(c) = c^{1-\sigma}/(1-\sigma) + \widehat{A}$, $\sigma \in (0, 1)$, $\kappa = y = 0$ and $b_0 = 0$. Define $A \equiv (1 + \rho)\widehat{A}$ and $R_t \equiv 1 + r_t$. Assume bequest constraints are binding. Conditions for this to be the case are discussed at the end of this section.

Individuals' problem From the Euler equation (7) and given the assumed utility function, it follows that

$$c_{2t+1} = (\rho R_{t+1})^{1/\sigma} c_{1t}, \text{ and} \quad (18)$$

$$U_t = \frac{c_{1t}^{1-\sigma}}{1-\sigma} + \rho \frac{c_{2t+1}^{1-\sigma}}{1-\sigma} + A = \frac{c_{1t}^{1-\sigma}}{1-\sigma} \left(1 + \rho^{1/\sigma} R_{t+1}^{1/\sigma-1}\right) + A \text{ for } t > 0. \quad (19)$$

Equation (18) together with the budget constraint, equation (6), can be used to solve for c_{1t} as

$$c_{1t} = \frac{w_t (1 - \delta n_t)}{1 + \rho^{1/\sigma} R_{t+1}^{1/\sigma-1}}. \quad (20)$$

On the other hand, individual savings, s_t , satisfy

$$s_t = w_t(1 - \delta n_t) - c_{1t} = \frac{w_t(1 - \delta n_t)}{1 + \rho^{-1/\sigma} R_{t+1}^{1-1/\sigma}}. \quad (21)$$

Regarding interior fertility choices, equation (9) can be used to rewrite (5) and (9) as

$$V_t = U_t + c_{1t}^{1-\sigma} \frac{1}{\epsilon^\Phi(n_t)} \frac{\delta w_t n_t}{c_{1t}}, \text{ and}$$

$$c_{1t}^{1-\sigma} \frac{\delta n_t w_t}{c_{1t}} = \epsilon^\Phi(n_t) \Phi(n_t) \left(\frac{U_{t+1}}{c_{1t+1}^{1-\sigma}} + \frac{1}{\epsilon^\Phi(n_{t+1})} \frac{\delta w_{t+1} n_{t+1}}{c_{1t+1}} \right) c_{1t+1}^{1-\sigma},$$

and using (19) and (20), this expression simplifies to

$$\begin{aligned} \frac{\delta n_t}{1 - \delta n_t} &= \Phi(n_t) \epsilon^\Phi(n_t) \left[\frac{1}{1 - \sigma} + \frac{1}{\epsilon^\Phi(n_{t+1})} \frac{\delta n_{t+1}}{1 - \delta n_{t+1}} + \frac{A}{c_{1t+1}^{1-\sigma} (1 + \rho^{1/\sigma} R_{t+2}^{1/\sigma-1})} \right] \\ &\times \left(\frac{c_{1t+1}}{c_{1t}} \right)^{1-\sigma} \frac{1 + \rho^{1/\sigma} R_{t+2}^{1/\sigma-1}}{1 + \rho^{1/\sigma} R_{t+1}^{1/\sigma-1}}. \end{aligned} \quad (22)$$

Firms Competitive firms produce output, y , using the technology $y = zF(k, l)$ where k is capital and l is labour. Let $zf(k) \equiv zF(k, 1)$ be production per unit of labour and assume $f(k) = k^\alpha$. Competition guarantees prices are equal to marginal products

$$R_t = \alpha z k_t^{\alpha-1} \text{ and } w_t = (1 - \alpha) z k_t^\alpha. \quad (23)$$

Demographics Let $N_t = N_{1t} + N_{2t}$ be total adult population at time t . The demographic structure satisfies the following conditions

$$\begin{aligned} N_{1t} &= n_{t-1} N_{2t}, & N_t &= (1 + n_{t-1}) N_{2t}, \\ \frac{N_{1t}}{N_t} &= \frac{n_{t-1}}{1 + n_{t-1}}, \text{ and } & \frac{N_{2t}}{N_t} &= \frac{1}{1 + n_{t-1}}. \end{aligned}$$

Aggregate resources Aggregate labour supply in the economy is $L_t = N_{1t}(1 - \delta n_t)$. Furthermore, aggregate capital in the economy satisfies $K_{t+1} = s_t N_{1t} = s_t N_{2t+1}$. Therefore

$$k_{t+1} = \frac{K_{t+1}}{L_{t+1}} = s_t \frac{N_{2t+1}}{N_{1t+1}} \frac{N_{1t+1}}{L_{t+1}} = s_t \frac{1}{n_t} \frac{1}{1 - \delta n_{t+1}}. \quad (24)$$

Steady state In steady state equation (22) simplifies to

$$\frac{\delta n^*}{1 - \delta n^*} \frac{1 - \Phi(n^*)}{\Phi(n^*)} = \epsilon^{\Phi(n^*)} \left[\frac{1}{\epsilon^u(c_1^*)} + \left(\frac{1 - (\rho R^*)^{1/\sigma-1}}{1 + \rho^{1/\sigma} (R^*)^{1/\sigma-1}} \right) \rho \hat{A} (c_1^*)^{\sigma-1} \right]. \quad (25)$$

This expression is just equation (14) when $\kappa = y = 0$ and $\rho R^* = 1$, case in which the results in Propositions 3 and 4 apply: specifically, equation (25) implies a negative fertility-consumption relationship only if $A > 0$ and $\sigma \in (0, 1)$. When $\rho R^* \neq 1$ then parental consumption has an additional second-order effect on the relative marginal benefit of having children determined by the last term of (25). If $\rho R^* < 1$ then this term is smaller for richer parents and therefore works toward generating a negative fertility-income relationship reinforcing the effects described in the previous section. Furthermore, according to (18), $\rho R^* < 1$ is also required to replicate a decreasing life-cycle profile of consumption upon retirement as is typically the case in the data. For these two reasons we focus the discussion that follows in the case $\rho R^* \leq 1$. In this case fertility is maximal for small enough consumption levels because $\epsilon^u(c_1^*)$ goes to zero, while for large enough consumption fertility converges to \tilde{n} , where \tilde{n} is the solution to

$$\frac{\delta \tilde{n}}{1 - \delta \tilde{n}} \frac{1 - \Phi(\tilde{n})}{\Phi(\tilde{n})} = \frac{\epsilon^{\Phi(\tilde{n})}}{1 - \sigma}. \quad (26)$$

LEMMA 2. *Let Assumption 1 (i) hold and $\rho R^* \leq 1$. Then the solution to (25) satisfies $\tilde{n} \leq n^* \leq N$.*

To fully solve for the steady state, the following two additional equations are used

$$1/R^* + \rho^{-1/\sigma} (R^*)^{-1/\sigma} = \frac{1 - \alpha}{\alpha} \frac{1}{n^*}, \text{ and} \quad (27)$$

$$c_1^* = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}} \frac{(1 - \delta n) (R^*)^{\frac{-\alpha}{1-\alpha}}}{1 + \rho^{1/\sigma} (R^*)^{1/\sigma-1}}. \quad (28)$$

These are steady-state versions of equations (24) and (20), obtained after using equations (23) and (21). Equation (27) describes a positive relationship between interest rates and fertility rates, say $R(n^*)$, while equation (28) shows that consumption is a positive function of total factor productivity, z , and a negative function of both interest rates and fertility. Since (26), (27) and (28) hold for large consumption levels, a key result that if TFP is sufficiently large then fertility and interest rates are independent of TFP while consumption fully responds to TFP. This is because (26) and

(27) can be used to solve for fertility and interest rates independently of TFP and then (28) can be used to solve for consumption. For intermediate values of TFP, and therefore intermediate values of consumption, higher TFP increases consumption but also reduces fertility, according to equation (25), and interest rates, according to equation (27), which further increases consumption. These results are summarized in the following proposition.

PROPOSITION 6. COMPARATIVE STATICS IN GENERAL EQUILIBRIUM. $\partial n^*/\partial z < 0$, $\partial R^*/\partial z < 0$ and $\partial c^*/\partial z > 0$ for z sufficiently large.

We now need to verify that binding bequest constraints and $\rho R^* < 1$ are in fact possible general equilibrium solutions. Rather than providing general conditions, we show some specific examples where this is in fact the case. According to (8), bequests constraints bind if steady state allocations and prices satisfy $1 > \beta(n^*)R^*$. Notice from equation (25) that if $\rho R^* \leq 1$ then $\delta > \Phi(n^*)/n^* = \beta(n^*)$ because the right hand side of (25) is larger than one, under Assumption A.1 (i). Therefore, given $\rho R^* \leq 1$, a sufficient condition for bequest constraints to bind is $1 > \delta R^*$ which is equivalent to the condition that children are a net benefit to parents: $w^*(1/R^* - \delta) > 0$. For example, if the cost of raising one child is less than 1/4 of parental time then the sufficient condition $1 > \delta R^*$ requires $R^* < 4$. Finally, if $\rho \leq \delta$ then the condition $1 > \delta R^*$ also guarantees $1 > \rho R^*$. Restriction $\rho \leq \delta$ means that children are sufficiently costly.

Finally, notice that the positive relationship between R^* and n described by (27) implies that R^* is highest when $n^* = N$. Therefore, a sufficient condition for $1 \geq \delta R^*$ is $1 \geq \delta R(N)$. Using (27), $R(N)$ is guaranteed to be below $1/\delta$ if the condition in the following proposition holds.

PROPOSITION 7. BINDING BEQUEST CONSTRAINTS. *Sufficient conditions for bequest constraints to bind are $\delta + (\delta/\rho)^{1/\sigma} < \frac{1-\alpha}{\alpha} \frac{1}{N}$ and $\rho \leq \delta$.*

Proposition 7 provides sufficient but not necessary conditions on parameters. Alternatively, suppose Φ is chosen so that optimal fertility is equal or below the replacement rate, $n^* \leq 1$, as is the case in many developed economies. Suppose further that $\rho = \delta$. Then the sufficient condition in Proposition 7 becomes $\delta + 1 < (1 - \alpha)/\alpha$. If $\alpha = 1/3$, as is the typical case considered in macro, then the condition becomes $\delta < 1$ which is always the case. These examples show that it is possible to construct a plausible general equilibrium model where fertility decreases with TFP, bequest constraints are binding and $1 \geq \rho R^*$.

4 Concluding comments

The analysis presented in this paper yields the following four main insights. First, explicitly considering the role of constraints to intergenerational transfers is essential in understanding the fertility-income relationship. While with no constraints fertility and income are unrelated, binding constraints recover the link between income and fertility. Bequest constraints bind in the model when the cost of raising children is below the present value of the children's income. Available data on the cost of raising children suggests that this is indeed the case. The theory of this paper is one in which children are a net financial benefit. In this case, even altruistic parents would like to be reimbursed by their own children for the cost of raising them, but this is unfeasible due to legal or other constraints. This mechanism is at the heart of the link between income and fertility.

The second insight of the analysis is that some additional restrictions are required for the correlation between income and fertility to be negative. Notably, the intergenerational elasticity of substitution must be larger than one. What this implies is that high wage parents find optimal to increase their own consumption and reduce fertility in spite of the fact that this creates a larger gap between their own consumption and that of a potential new born child. The idea of intergenerational substitution is novel, and the estimation of its value becomes an important avenue for future research.

The third insight of this paper is that in general equilibrium the fertility-income relationship translates into a relationship between the level of total factor productivity (TFP) and fertility. Such a link cannot be obtained in models in which unrestricted intergenerational transfers are allowed. This link suggests that the same forces used by others researchers to explain cross-country income differences (e.g., Hall and Jones, 1999, among others), namely differences in TFP, can also explain why fertility declines with income in modern times.

As a last insight, our paper provides a link between purely altruistic and some non-altruistic models. Specifically, when bequest constraints bind in purely altruistic models, the determinants of fertility may be similar to those in which the parent cares about the number and/ or quality of the children. The reason is that in this case fertility is determined by a comparison of marginal benefits and marginal costs of having children. Therefore, elements such as parental income and the technology of raising children become relevant.

Our paper provides a framework of analysis in which a number of other intergenerational issues

can be examined. OLG models with altruism, endogenous fertility and constraints to intergenerational transfers are not only consistent with the fertility-income relationship, but combine the necessary elements to study policies that affect redistribution across generations, as well as intergenerational transmission of inequality. These are questions we are analyzing in on-going research.

A Appendix

A.1 Boundedness and transversality conditions

Following Theorem 4.3 in Stockey and Lucas (1989), for the principle of optimality to hold the condition $\lim_{T \rightarrow \infty} \Pi_{t=0}^{T-1} \Phi(n_t) V(b_{T+1}) = 0$ for all b_T must be satisfied. A more familiar condition is obtained when fertility is constant. In this case the boundedness condition simplifies to $\lim_{T \rightarrow \infty} \Phi(n)^T V(b_T) = 0$. When bequests constraints bind, as is the case in model, then the boundedness condition above simplifies to $\lim_{T \rightarrow \infty} \Pi_{t=0}^{T-1} \Phi(n_t) = 0$. This condition is satisfied due to assumed restriction $\Phi(n) \leq \Phi(N) < 1$. For the case of unrestricted bequests and competitive markets, in a model with endogenous population the usual no-Ponzi game condition (NPG) becomes

$$\lim_{T \rightarrow \infty} \frac{B_{T+1}}{(1+r)^T} \geq 0, \quad (\text{A.1})$$

where B_{T+1} are total assets of the dynasty. B can be written as $B_{T+1} = b_{t+1} L_{t+1}$ where $L_{T+1} = \Pi_{t=0}^T n_t$ is family size at $T + 1$. At the optimum, (A.1) holds with equality. In that case, this condition can also be written as the transversality condition for a planner as follows. Using the intergenerational Euler Equation of the problem, one obtains

$$\begin{aligned} u'(c_0) &= (1+r) \beta(n_0) u'(c_1) = (1+r)^2 \beta(n_0) \beta(n_1) u'(c_2) \\ &= (1+r)^T u'(c_T) \Pi_{t=0}^T \beta(n_t). \end{aligned}$$

Solving for $(1+r)^T$ and substituting the result into (A.1) results in

$$\begin{aligned} 0 &= \lim_{T \rightarrow \infty} b_{T+1} L_{T+1} \frac{u'(c_T) \Pi_{t=0}^T \beta(n_t)}{u'(c_0)} = \lim_{T \rightarrow \infty} b_{T+1} \frac{u'(c_T) \Pi_{t=0}^T n_t \beta(n_t)}{u'(c_0)} \\ &= \frac{1}{u'(c_0)} \lim_{T \rightarrow \infty} [\Pi_{t=0}^T \Phi(n_t)] b_{T+1} u'(c_T). \end{aligned}$$

A.2 Assumptions for existence of an interior fertility solution

Some restrictions are needed in order to guarantee a well-behaved problem. For this purpose it is convenient to characterize the relevant functions in terms of their elasticities. We use the notation $\epsilon^F(x)$ to denote the elasticity of function F with respect to variable x .

ASSUMPTION A.1. UTILITY AND ALTRUISM. Let $c(n) \equiv w + y - n(\delta w + \kappa)$. Functions Φ and u satisfy: (i) $\epsilon^\Phi(n) > \epsilon^u(c)$; (ii) $1 > \beta(N)(1 + r)$; (iii) $\epsilon^u(w + y)(\delta w + \kappa) < \Phi'(0)(w + y)$; (iv) $\epsilon^u(c(N))(\delta w + \kappa) > \frac{\Phi'(N)}{1 - \Phi(N)}c(N)$; (v) and $\frac{\Phi'(n)}{1 - \Phi(n)}$ strictly decreases with n for $n \in [0, N]$.

Parts (i) and (ii) of Assumption A.1 are needed for the existence of an interior solution for fertility when bequests constraints are not binding. In particular, condition (i) is needed to prevent $n = 0$ to be the optimal solution as discussed in Becker and Barro (1988). The remaining conditions are used to guarantee a unique interior solution for fertility when bequest constraints are binding. Condition (iii) ensures that the marginal cost of the first dn children is smaller than the marginal benefit; condition (iv) guarantees that marginal cost of N children is higher than its marginal benefit; and condition (v) guarantees a single crossing between marginal costs and marginal benefits.²⁵ Overall, these conditions are mild and require some minimum degree of altruism, diminishing degree of altruism and a mild bound on the double-elasticity of utility with respect to consumption. The last restriction is trivially satisfied when function u is isoelastic. Examples of functions Φ satisfying condition (v) as well as the restriction $0 \leq \Phi(n) < 1$ for all $n \geq 0$ are $\Phi(n) = \alpha(1 - e^{-\mu n})$ and $\Phi(n) = (\alpha\mu n)/(1 + \mu n)$ for $0 < \alpha < 1$ and $\mu > 0$.²⁶

A.3 Proof of Proposition 1

The steady state is characterized by equations

$$c^* = \frac{1}{1 + \rho} [w + y - n^*(\delta w + \kappa)], \text{ and}$$

$$c^* = \frac{\epsilon^u(c^*)}{1 + \rho} (\delta w + \kappa) m(n^*),$$

²⁵Some intuition for condition (v) is obtained if $\Phi(n)$ is interpreted as the survival probability of n children. In this case $\frac{\Phi'(n)}{1 - \Phi(n)}$ is the survival probability conditional on being "unborn," and condition (v) requires it to be diminishing.

²⁶The function $\Phi(n) = \alpha n^\theta$ used by Becker and Barro (1988) needs an added restriction on the maximum number of children in order to satisfy Assumption A.1 (v).

where $m(n^*) = (1 - \Phi(n^*))/\Phi'(n^*)$ is an increasing function by Assumption A.1 (v). Linearizing these equations around the steady state yields

$$dc = \frac{1}{1+\rho} [(1 - \delta n^*) dw + dy - (\delta w + \kappa) dn], \quad (\text{A.2})$$

$$dc = c^* \left(\frac{(\epsilon^u)'}{\epsilon^u} dc + \frac{\delta}{\delta w + \kappa} dw + \frac{m'}{m} dn \right). \quad (\text{A.3})$$

Equation (A.3) can be simplified to

$$dc = \frac{c^*}{1 - \epsilon^{\epsilon^u}} \left[\frac{m'}{m} dn + \frac{\delta}{\delta w + \kappa} dw \right], \quad (\text{A.4})$$

where $\epsilon^{\epsilon^u} = ((\epsilon^u)'/c^*)/\epsilon^u$. Plugging (A.4) into (A.2),

$$c^* \frac{m'}{m} dn + c^* \frac{\delta}{\delta w + \kappa} dw = \frac{1 - \epsilon^{\epsilon^u}}{1 + \rho} [(1 - \delta n^*) dw + dy - (\delta w + \kappa) dn],$$

and collecting terms,

$$\underbrace{\left(c^* \frac{m'}{m} + \frac{1 - \epsilon^{\epsilon^u}}{1 + \rho} (\delta w + \kappa) \right)}_{+} dn = \underbrace{\left(\frac{1 - \epsilon^{\epsilon^u}}{1 + \rho} (1 - \delta n^*) - c^* \frac{\delta}{\delta w + \kappa} \right)}_{?} dw + \underbrace{\frac{1 - \epsilon^{\epsilon^u}}{1 + \rho}}_{+} dy.$$

Finally, solving for dn ,

$$dn = \frac{\frac{1 - \epsilon^{\epsilon^u}}{1 + \rho} (1 - \delta n^*) - c^* \frac{\delta}{\delta w + \kappa}}{\underbrace{c^* \frac{m'}{m} + \frac{1 - \epsilon^{\epsilon^u}}{1 + \rho} (\delta w + \kappa)}_{?}} dw + \frac{\frac{1 - \epsilon^{\epsilon^u}}{1 + \rho}}{\underbrace{c^* \frac{m'}{m} + \frac{1 - \epsilon^{\epsilon^u}}{1 + \rho} (\delta w + \kappa)}_{+}} dy, \quad (\text{A.5})$$

which implies that around the steady state n is increasing in y , while the effect of w on n is undetermined as stated in Proposition 1 in the text. In order to obtain $\partial n^*/\partial w < 0$ one needs

$$\frac{1 - \epsilon^{\epsilon^u}}{1 + \rho} (1 - \delta n^*) < c^* \frac{\delta}{\delta w + \kappa} = \frac{1}{1 + \rho} [w + y - n^* (\delta w + \kappa)] \frac{\delta}{\delta w + \kappa}$$

or

$$(1 - \epsilon^{\epsilon^u}) (1 - \delta n^*) \frac{\delta w + \kappa}{\delta} + n^* (\delta w + \kappa) < w + y.$$

A sufficient condition for $\partial n^*/\partial w < 0$ is:

$$(1 - \epsilon^u) (1 - \delta n^*) \frac{\delta w + \kappa}{\delta} + N(\delta w + \kappa) < w + y.$$

Next, replacing the expression for dn in (A.5) into (A.2) and collecting terms yields

$$dc = \frac{1}{1 + \rho} \left[\underbrace{\left(\frac{\frac{m'}{m} + \delta}{\frac{m'}{m} + \frac{1 - \epsilon^u}{1 + \rho} \frac{\delta w + \kappa}{c^*}} \right)}_{+} dw + \underbrace{\left(\frac{\frac{c^* m'}{m}}{c^* \frac{m'}{m} + \frac{1 - \epsilon^u}{1 + \rho} (\delta w + \kappa)} \right)}_{+} dy \right],$$

which implies that around the steady state c is increasing in both y and w as stated in Proposition 1.

A.4 Proof of Proposition 3

Suppose $w \rightarrow \infty$. From equation (12) it follows that $c \rightarrow \infty$ since $n^* \leq N$. Furthermore, notice that

$$\epsilon^u(c^*) = \frac{1}{\frac{1}{1 - \sigma} + A(c^*)^{\sigma - 1}} = \frac{1 - \sigma}{1 + (1 - \sigma) A c^{\sigma - 1}} > 0.$$

Consider first the case $\sigma > 1$. In that case, $\lim_{w \rightarrow \infty} \epsilon^u(c^*) = 0$. Thus, the limit of the relative marginal cost in equation (14) is bounded, while the relative marginal benefit is infinity any $n \leq N$. As a result, maximum fertility, $n^* = N$, is optimal. For the case $\sigma < 1$, $\lim_{w \rightarrow \infty} \epsilon^u(c^*) = 1 - \sigma$ and equation (14) can be written as (15).

A.5 Proof of Lemma 1

First, write (13) as

$$\ln(\delta w + \kappa) - \sigma \ln c^* - \ln \left(\frac{1}{1 - \sigma} (c^*)^{1 - \sigma} + A \right) = \ln \Phi'(n^*) - \ln(1 - \Phi(n^*)) + \ln(1 + \rho).$$

Totally differentiating this equation around the steady state one obtains

$$\frac{\delta dw}{\delta w + \kappa} - \sigma \frac{dc}{c^*} - \frac{(c^*)^{-\sigma} dc}{\frac{1}{1 - \sigma} (c^*)^{1 - \sigma} + A} = \frac{\Phi''}{\Phi'} dn + \frac{\Phi'}{1 - \Phi} dn,$$

or

$$\frac{\delta dw}{\delta w + \kappa} - \Psi(c^*) \frac{dc}{c^*} = \left[\frac{\Phi''}{\Phi'} + \frac{\Phi'}{1 - \Phi} \right] dn,$$

where

$$\Psi(c^*) = \sigma + \frac{(c^*)^{1-\sigma}}{\frac{1}{1-\sigma} (c^*)^{1-\sigma} + A} = \sigma + \epsilon^u(c^*) > 0.$$

Since (12) implies $(1 + \rho) dc = -(\delta w + \kappa) dn + (1 - \delta n^*) dw$, then the previous equation can be written as

$$\left[\frac{\delta}{\delta w + \kappa} - \frac{\Psi}{c^*(1 + \rho)} (1 - \delta n^*) \right] dw = \left[\frac{\Phi''}{\Phi'} + \frac{\Phi'}{1 - \Phi} - (\delta w + \kappa) \frac{\Psi}{c^*(1 + \rho)} \right] dn.$$

Next, notice from (13) that

$$\frac{\Phi'(n^*)}{1 - \Phi(n^*)} = \frac{\delta w + \kappa}{c^*(1 + \rho)} (\Psi - \sigma),$$

and thus

$$\frac{dn}{dw} = \frac{-\frac{\delta}{\delta w + \kappa} + \frac{\Psi}{c^*(1 + \rho)} (1 - \delta n^*)}{-\frac{\Phi''}{\Phi'} + \frac{\delta w + \kappa}{c^*(1 + \rho)} \sigma}.$$

Then the denominator is always positive and $dn/dw < 0$ if and only if

$$-\frac{\delta}{\delta w + \kappa} + \frac{\Psi}{c^*(1 + \rho)} (1 - \delta n^*) < 0,$$

or

$$1 + \frac{y}{w} > [\Psi(c^*)(1 - \delta n^*) + \delta n^*] \left(1 + \frac{\kappa}{w} \right),$$

which corresponds to (16) in the text.

Iowa State University

University of Pittsburgh

Submitted: 28 March 2013

Accepted:

References

- [1] Abel, A. (1987). ‘Operative gift and bequest motives’, *American Economic Review*, vol. 77(5), pp. 1037-1047.
- [2] Altonji, J., Hayashi, F. and Kotlikoff, L. (1997). ‘Parental altruism and inter vivos transfers: theory and evidence’, *Journal of Political Economy*, vol. 105(6), pp. 1121-1166.
- [3] Alvarez, F. (1999). ‘Social mobility: the Barro-Becker children meet the Laitner-Loury dynasties’, *Review of Economic Dynamics*, vol. 2(1), pp. 65-103.
- [4] Ashraf, Q. and Galor, O. (2011). ‘Dynamics and stagnation in the Malthusian epoch’, *American Economic Review*, vol. 101(5), pp. 2003-2041.
- [5] Barro, R. and Becker, G. (1989). ‘Fertility choice in a model of economic growth’, *Econometrica*, vol. 57(2), pp. 481-501.
- [6] Barro, R. and Sala-i-Martin, X. (2004). *Economic Growth*, Cambridge: MIT Press.
- [7] Becker, G. (1960). ‘An economic analysis of fertility’, in (National Bureau of Economic Research), *Demographic and Economic Change in Developed Countries*, pp. 209-240. New York: Columbia University Press.
- [8] Becker, G. and Barro, R. (1988). ‘A reformulation of the economic theory of fertility’, *Quarterly Journal of Economics*, vol. 103 (1), pp. 1-25.
- [9] Becker, G., Murphy, K. and Tamura, R. (1990). ‘Human capital, fertility, and economic growth’, *Journal of Political Economy*, vol. 98(5), pp. S12–S37.
- [10] Cordoba, J.C. and Ripoll, M. (2011). ‘A contribution to the economic theory of fertility’, Working Paper, Iowa State University.
- [11] Cordoba, J.C. and Ripoll, M. (2013). ‘What explains schooling differences across countries?’, *Journal of Monetary Economics*, vol. 60(2), pp. 184-202.
- [12] Cordoba, J.C., Liu, X. and Ripoll, M. (2014). ‘Fertility, social mobility and long run inequality’, Working Paper, Iowa State University.

- [13] Cordoba, J.C. and Liu, X. (2014). ‘Poverty and the optimality of family planning policies’, Working Paper, Iowa State University.
- [14] De la Croix, D. and Vander Donckt, M. (2010). ‘Would empowering women initiate the demographic transition in least developed countries?’ *Journal of Human Capital*, vol. 4 (2), pp. 85-129.
- [15] Doepke, M. (2004). ‘Accounting for fertility decline during the transition to growth’, *Journal of Economic Growth*, vol. 9(3), pp. 347–383.
- [16] Folbre, N. (2008). *Valuing Children: Rethinking the Economics of the Family*, Cambridge: Harvard University Press.
- [17] Galor, O., Moav O. and Vollrath, D. (2009). ‘Inequality in land ownership, the emergence of human-capital promoting institutions, and the great divergence’, *Review of Economic Studies*, vol. 76 (1), pp. 143-179.
- [18] Galor, O. and Weil, D. (2000). ‘Population, technology, and growth: from Malthusian stagnation to the demographic transition and beyond’, *American Economic Review*, vol. 90(4), pp. 806–828.
- [19] Greenwood, J. and Seshadri, A. (2002). ‘The U.S. demographic transition’, *American Economic Review Papers and Proceedings*, vol. 92(2), pp. 153–159.
- [20] Guryan, J., Hurst, E. and Kearney, M. (2008). ‘Parental education and parental time with children’, *Journal of Economic Perspectives*, vol. 22(3), pp. 23-46.
- [21] Hall, R. and Jones, C. (2007). ‘The value of life and the rise in health care spending’, *Quarterly Journal of Economics*, vol. 122(1), pp. 39-72.
- [22] Hall, R. and Jones, C. (1999). ‘Why do some countries produce so much more output per worker than others?’, *Quarterly Journal of Economics*, vol. 114(1), pp. 83-116.
- [23] Hurd, M. and Smith, J. (2001). ‘Anticipated and actual bequest’, in (D. Wise, ed.), *Themes in the Economics of Aging*, pp. 357-392. Chicago: University of Chicago Press.
- [24] Jones, C. (2001). ‘Was an Industrial Revolution inevitable? Economic growth over the very long run’, *Advances in Macroeconomics*, vol. 1(2), pp. 1-45.

- [25] Jones, L., Schoonbroodt, A. and Tertilt, M. (2011). ‘Fertility theories: can they explain the negative fertility-income relationship?’, in (J. Shoven, ed.), *Demography and the Economy*, pp. 43-100. Chicago: University of Chicago Press.
- [26] Jones, L. and Schoonbroodt, A. (2014). ‘Baby busts and baby booms: the fertility response to shocks in dynastic models’, Working Paper, University of Iowa.
- [27] Jones, L. and Schoonbroodt, A. (2010). ‘Complements versus substitutes and trends in fertility choice in dynastic models’, *International Economic Review*, vol. 51(3), pp. 671-699.
- [28] Jones, L. and Schoonbroodt, A. (2009). ‘Complements versus substitutes and trends in fertility choice in dynastic models: appendix’, Working Paper, University of Iowa.
- [29] Jones, L. and Tertilt, M. (2008). ‘An economic history of fertility in the United States: 1826-1960’, in: (P. Rupert, ed.), *Frontiers of Family Economics*, vol. 1, pp. 165-230. Bingley: Emerald Group Publishing Limited.
- [30] Kalemli-Ozcan, S. (2003). ‘A stochastic model of mortality, fertility and human capital investment’, *Journal of Development Economics*, vol. 70(1), pp. 103-118.
- [31] Kambourov, G. and Manovskii, I. (2009). ‘Accounting for the changing life-cycle profile of earnings’, Working Paper, University of Pennsylvania.
- [32] Kaur, H. (2000). ‘Impact of income and education on fertility’, *Journal of Family Welfare*, vol. 46(1), pp. 70-76.
- [33] Lam, D. (1986). ‘The dynamics of population growth, differential fertility, and inequality’, *American Economic Review*, vol. 76(5), pp. 1103-1116.
- [34] Lapan, H. and Enders, W. (1990). ‘Endogenous fertility, Ricardian equivalence and debt management policy’, *Journal of Public Economics*, vol. 41(2), pp. 227-248.
- [35] Schoonbroodt, A. and Tertilt, M. (2010). ‘Who owns children and does it matter?’, NBER Working Paper # 15663.
- [36] Schoonbroodt, A. and Tertilt, M. (2014). ‘Property rights and efficiency in OLG models with endogenous fertility’, *Journal of Economic Theory*, vol. 150(C), pp. 551-583.

- [37] Stockey, N. and Lucas, R. (1989). *Recursive Methods in Economic Dynamics*, Cambridge: Harvard University Press.
- [38] Tamura, R. (2006). ‘Human capital and economic development’, *Journal of Development Economics*, vol. 79(1), pp. 26-72.
- [39] USDA. (2012). ‘Expenditures on children by families, 2011’, Miscellaneous Publication Number 1528-2011.
- [40] Weil, P. (1987). ‘Love thy children: reflections on the Barro debt neutrality theorem’, *Journal of Monetary Economics*, vol. 19(3), pp. 377-391.

Table 1
Cost of raising children ages 0 to 17 - United States
 Present value in 2011 U\$

Family income group	Goods cost USDA (2012)	Time cost		Total cost	
		Scenario 1: 60% of total cost	Scenario 2: 75% of total cost	Scenario 1	Scenario 2
Low income	148,962	223,443	446,886	372,405	595,848
Middle income	206,709	310,063	620,127	516,772	826,836
High income	342,759	514,138	1,028,227	856,897	1,371,036

Notes: Family income groups correspond to the categories in USDA (2012): low refers to families with before-tax income below \$59,410 in 2011; middle between \$59,410 and \$102,870; and high above the latter. Goods costs correspond to the USDA (2012) projected direct parental expenses (housing, food, transportation, health care, clothing, child care and private education) made on a child born in 2011 from age 0 to 17, assuming an inflation rate of 2.55%. Costs are measured for a family with two parents and two children. Present value is computed using a discount rate of 1.5%. Scenario 1 corresponds to the case in which parents spend 21 hours per week in child care (Guryan *et al.*, 2008), while under scenario 2 parents spend 41 hours per week (Folbre, 2008). These time costs are imputed using Folbre's (2008) estimates of the share of time costs on total costs of raising children on her Table 7.3 (p. 133). Hours are valued at the median wage.

Table 2
Net financial benefit of raising children ages 0 to 17 - United States
 Present value in 2011 U\$

Family income group	Future child's income	Total costs of raising child		Net financial benefit	
		Scenario 1	Scenario 2	Scenario 1	Scenario 2
Low income	661,529	372,405	595,848	289,124	65,681
Middle income	1,463,970	516,772	826,836	947,198	637,134
High income	2,281,206	856,879	1,371,036	1,424,309	910,170

Notes: Future child's income is computed assuming the child works from age 18 to age 65. The life-cycle income of the child is computed using the polynomial in Kambourov and Manovskii (2009) and assuming the child at age 28 has the same income as the parent at age 28, adjusted for a real growth rate of 1% per year. Family income when the child is born in 2011 is computed as a point within the USDA (2012) income group brackets: low income is \$36,665; middle income is \$81,140 and high income is assumed to be \$126,435. Present values are computed using a discount rate of 1.5%. Scenarios 1 and 2 of the total cost of raising a child are extracted from Table 1. The net financial benefit of raising a child is the difference between the present value of the child's future income and the present value of the total costs of raising the child.