Risk Aversion and the Value of Life

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Abstract

We show that state non-separable preferences à la Epstein-Zin-Weil (EZW) provide a tractable and flexible framework to study the economics of health and longevity. This utility representation: *(i)* admits a preference for timing of resolution of uncertainty regarding mortality risks; *(ii)* links the marginal valuation of survival to the level of survival; *(iii)* can preserve homotheticity even for low degrees of intertemporal substitution without generating implausible predictions regarding the value of life; and *(iv)* adds needed flexibility to account for the empirical evidence on the value of life. We illustrate the implications of EZW preferences for the economic value of observed differences in life expectancy across countries and over time, and for the value of life over the life cycle.

Key words: life expectancy, value of statistical life, Epstein-Zin-Weil preferences, welfare, AIDS

JEL Codes: I15, J17

1 INTRODUCTION

The degree of aversion to mortality risk is central to assess the economic value of medical research, as well as health, environmental, and various policy interventions affecting mortality rates. Micro-founded dynamic models with mortality risk have become increasingly popular to study issues of health and longevity. Recent examples include Murphy and Topel (2006) and Hall and Jones (2007), who study the economic value of health improvements and the reasons for the secular increase in health spending in the US; and Becker, Philipson and Soares (2005) and Jones and Klenow (2011), who estimate the economic gains associated to lower mortality rates around the world. A common feature of this literature is the use of the expected utility model. Although this type of framework has been used to study a variety of economic issues, it is not clear that it is also desirable in the study of longevity issues.

This paper discusses properties of preference representations that are desirable to study mortality issues. In particular, we examine the properties of a state non-separable utility representation along the lines proposed by Kreps and Porteus (1978), Epstein and Zin (1989, 1991) and Weil

(1990) –EZW henceforth. We show that this representation provides a natural, tractable and flexible framework to study aversion to mortality risk. By separating risk aversion from intertemporal substitution, the EZW framework displays flexibility in four dimensions relevant to the economics of health and longevity: *(i)* it admits a preference for the timing of resolution of uncertainty regarding mortality risks; *(ii)* it allows the marginal valuation of survival to depend on the level of survival; *(iii)* it can preserve homotheticity even for low degrees of intertemporal substitution without generating implausible predictions regarding the value of life; and *(iv)* it provides superior flexibility to match the empirical evidence on the value of life.

First, available empirical and experimental evidence indicates that individuals are not indifferent to the timing of the resolution of uncertainty.[1] In the case of mortality risk, studies looking at uptake rates of genetic testing for fatal illnesses show that many individuals choose not to learn the information provided by these tests (Oster *et al.*, 2013). Related studies report that individuals at risk avoid testing because for them "termination comes not at the moment of death but at the moment of diagnosis" (Wexler, 1979, p. 199). As Epstein *et al.* (2013) indicate, given that the information from genetic testing has clear instrumental value, the evidence of low uptake rates is at least suggestive of a negative psychic benefit of early resolution. The EZW utility model admits a preference for early or late resolution of uncertainty and therefore offers a natural benchmark for studying mortality risk.

Second, the imputed economic benefit of any intervention that changes mortality rates (e.g. public health, road safety, medical procedures, a peace treaty) depends on whether or not utility is linear in probabilities. Linearity implies that an individual's willingness to pay for the intervention is the same regardless of whether the probability of surviving without it is 5% or 95%. Evidence, however, suggests that non-linearities may be important for the economics of health. For instance, Becker *et al.* (2007) argue that a decreasing marginal benefit of survival helps rationalize why the elderly are willing to pay nontrivial amounts to extend their short remaining life span. EZW preferences are generally non-linear in survival probabilities. By breaking linearity, EZW utility allows the marginal valuation of survival to decrease or increase with the level of survival.

Third, a useful model of longevity should provide plausible theoretical predictions regarding the value of life. For the class of (time-and-state separable) expected utility models this generally requires utility to be non-homothetic. Specifically, for the most common case in which the elasticity of intertemporal substitution is less than one, adding a minimum consumption level is unavoidable in order to obtain plausible values of life (Murphy and Topel, 2006; Hall and Jones 2007; Jones and Klenow, 2011). At least two issues arise when utility is non-homothetic. First, life-or-death gambles would be welfare enhancing because individuals whose consumption is below the minimum would be willing to pay to enter a Russian-roulette type of lottery that increases the chances of dying, in exchange for higher consumption in the event of surviving (Rosen, 1988). But such welfare-enhancing life-or-death gambles are hardly observed in practice. Second, non-homotheticity automatically introduces an income effect in the willingness to pay for life, implying particularly low values for individuals whose consumption is close to the minimum. In contrast, EZW preferences

[1] See Brown and Kim (2014) for a summary of the evidence regarding financial risk.

can easily handle the low intertemporal substitution case while maintaining homotheticity, and without generating implausible predictions regarding the value of life. This is possible because the value of life in EZW preferences is not strongly linked to the elasticity of intertemporal substitution, but to the coefficient of risk aversion. This leads to our last point.

Fourth, accounting for the empirical evidence of the value of life is central when evaluating the economic benefits of policies affecting mortality risk. EZW utility offers a more flexible framework for this purpose. In particular, EZW preferences are a parsimonious generalization of expected utility, with one more parameter: the coefficient of risk aversion. This parameter is conceptually appealing to the health and longevity literature because it measures the degree of aversion to mortality risk. Within the EZW framework, the coefficient of risk aversion is the natural parameter determining the economic value of life. Relative to prevalent expected utility models, this added flexibility diminishes the influence of non-homothetic parameters, and facilitates matching the evidence on the value of life for all income levels. In fact, the literature has recognized that an issue with expected utility models is that the size of minimum consumption affects the willingness to pay for life in a non-trivial manner (Murphy and Topel, 2006; Jones and Klenow, 2011). More importantly, by construction, individuals whose consumption is close to the minimum would exhibit particularly low willingness to pay for life. However, available evidence on the value of life in poor countries does not support this prediction (Viscusi and Aldy, 2003).

We illustrate the quantitative predictions of EZW preferences in two different contexts: comparisons of well-being across countries, and the value of life over the life cycle for the US. Regarding cross-country comparisons, we assess the economic value of longevity changes for the period 1970-2005 in a panel of 144 countries. For this purpose, we calibrate a version of our EZW model that abstracts from life-cycle features. We compare our results to those of Becker *et al.* (2005), who analyze an otherwise similar but expected utility model. We find that with EZW utility the value of life in poor countries is larger relative to Becker *et al.* (2005), while it is similar for richer countries. This is the result of two forces. First, the calibration implies diminishing returns to survival in the EZW model, so all else equal, life is more valuable in countries with shorter life spans. Second, EZW utility dampens the asymmetric effect of minimum consumption on the valuation of life in poor relative to rich countries. For instance, the ratio of value-of-life to income is sharply increasing in income in Becker *et al.* (2005) because consumption in poorer countries is close to the calibrated minimum. Available cross-country estimates of the value of life in poor countries do not support this sharply increasing pattern (Viscusi and Aldy, 2003).

We also use the calibrated model to calculate full measures of income that include the gains in longevity between 1970 and 2005. The EZW model penalizes losses and favors gains much more than existing models. Similarly, the EZW model implies that adjusting per-capita income to reflect cross-country differences in life expectancy in 2005 results in significantly larger world inequality. Finally, we assess the welfare effects of positive events like the end of wars and devastating events like the AIDS pandemic. For this purpose we compute full measures of income for 1990 and 2005, the relevant dates for the AIDS epidemic. We again find that the economic value of the loss in life due to AIDS and of the gain in life due to the end of wars is significantly higher than previous estimates.

Our second application examines the quantitative implications of EZW preferences for the value of life at different ages. For this purpose we calibrate a version of the EZW model that includes life-cycle features, and compare our results to those of Murphy and Topel (2006) who perform a similar exercise using a standard expected utility model. We find that for the average full-time male worker in the US, the value of life for ages 20 to 40, and after age 80, is higher than what Murphy and Topel (2006) report. The differences arise because individuals have lower income at the beginning and at the end of their life cycle, so consumption at those ages is closer to the minimum, an effect not present in our model. In addition, the calibrated EZW model implies that all else equal, life is more valuable for those with shorter life spans, the elderly. Last, for low-income individuals in the US, the value of life over the whole life cycle is larger under EZW utility than with expected utility. In sum, EZW utility has distinct quantitative predictions for young adults, the old, and low-income individuals.

The remainder of the paper is organized as follows. Section 2 discusses the properties of preferences that are desirable to study longevity issues. Section 3 presents a state non-separable EZW model and derives its implications for the value of life extensions. Section 4 illustrates the quantitative predictions of the EZW model both across countries, and across ages in the US. Section 5 discusses further implications of the analysis, and Section 6 concludes.

2 PREFERENCES AND MORTALITY RISK

The properties of preferences regarding consumption risk have been studied extensively, but properties associated to mortality risk remain largely unexplored (Rosen, 1988). In order to discuss some of these properties, consider a state non-separable representation of preferences. The parametric class of Epstein and Zin (1989) and Weil (1990) is particularly convenient. In fact, while there are other state non-separable specifications, EZW utility is one of the most popular departures from expected utility (EU) in macroeconomics. There are various reasons for this popularity: EZW preferences are recursive and time consistent; they are also parsimonious and tractable; and they disentangle intertemporal substitution from risk aversion, concepts that are described by distinct and constant parameters. In addition, standard EU can be easily obtained as a special case of EZW preferences, facilitating the comparison between the two representations.

Consider an individual of age t who consumes z_t (≥ 0) at age t, and survives to age $t+1$ with probability π_t. Assume the utility of remaining life is described by

$$W_t = (1-\gamma)^{-1} \left[z_t^{1-\sigma} + \beta [(1-\gamma)\, E_t \widetilde{W}_{t+1}]^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1-\gamma}{1-\sigma}}, \tag{1}$$

where $1 > \beta > 0$ is a discount factor, $1/\sigma$ is the elasticity of intertemporal substitution ($\sigma \geq 0$), parameter $\gamma \geq 0$ governs mortality risk aversion, and $E_t\widetilde{W}_{t+1}$ is the expected utility over the life-or-death lottery. Equation (1) is a parametric version of Kreps and Porteus' (1978) preferences proposed by Epstein and Zin (1989). The separation between σ and γ constitutes the main feature of EZW preferences, and EU corresponds to the special case $\sigma = \gamma$. Letting $\underline{W}$ be the individual's perceived utility upon death, we can write equation (1) as

$$W_t = (1-\gamma)^{-1} \left\{ z_t^{1-\sigma} + \beta \left[(1-\gamma) \left(\pi_t W_{t+1} + (1-\pi_t) \underline{W} \right) \right]^{\frac{1-\sigma}{1-\gamma}} \right\}^{\frac{1-\gamma}{1-\sigma}}, \tag{2}$$

and obtain the EU formulation when $\sigma = \gamma$ as

$$W_t^{EU} = \frac{z_t^{1-\sigma}}{1-\sigma} + \beta [\pi_t W_{t+1}^{EU} + (1-\pi_t) \underline{W}]. \tag{3}$$

Notice that W_t is (generally) negative when $\gamma > 1$ and non-negative when $\gamma \in [0,1)$. The following is a convenient monotonic transformation of (2), also used by Epstein and Zin (1989), that preserves the preference ordering and guarantees strictly positive utilities. Defining $V \equiv [(1-\gamma) W]^{\frac{1}{1-\gamma}}$, rewrite (2) as

$$V_t = \left[z_t^{1-\sigma} + \beta [\pi_t V_{t+1}^{1-\gamma} + (1-\pi_t) \underline{V}^{1-\gamma}]^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1}{1-\sigma}}, \tag{4}$$

which in the case $\sigma = \gamma$ reduces to

$$V_t^{EU} = \left[z_t^{1-\sigma} + \beta [\pi_t \left(V_{t+1}^{EU} \right)^{1-\sigma} + (1-\pi_t) \underline{V}^{1-\sigma}] \right]^{\frac{1}{1-\sigma}}, \tag{5}$$

where $\underline{V} \equiv [(1-\gamma) \underline{W}]^{\frac{1}{1-\gamma}} \geq 0$ is the transformed perceived utility upon death.[2]

We now discuss four aspects of preferences relevant to issues of mortality risk: *(i)* the preference for the timing of resolution of uncertainty; *(ii)* the marginal valuation of survival; *(iii)* homotheticity with low degree of intertemporal substitution; and *(iv)* the added flexibility useful for empirical purposes.

2.1 *Preference for the timing of resolution of uncertainty*

The thought experiment underlying the notion of preference for timing asks the following question: "how much would you pay to have your lifetime risk resolved next month, keeping in mind that you cannot use that information" (Epstein *et al.*, 2013, p. 12). The preference for the timing of resolution of uncertainty refers thus to the "psychic" effects of the resolution, not to the decision value of the information to the individual or to a planning advantage. If actions could be taken after receiving information, even the standard expected utility model would exhibit a preference for early resolution of uncertainty. But when no actions can be taken, the information revealed upon resolution of uncertainty only has a psychic effect (Strzalecki, 2013).

Kreps and Porteus (1978, Theorem 3) show that agents exhibit a preference for early (late)

[2] We follow Epstein and Zin (1989) in referring to equation (5) as the EU case since it describes the same preference ordering as (3).

resolution of uncertainty depending on whether lifetime utility is convex (concave) in $E_t\widetilde{W}_{t+1}$. For the case described in (1), early (late) resolution of uncertainty is preferred if $\sigma < \gamma$ ($\sigma > \gamma$), while indifference requires $\sigma = \gamma$, the EU case. To gain some intuition for this result, notice that parameter σ governs aversion to temporal deterministic fluctuations of consumption, while γ governs aversion to atemporal random fluctuations. Given that under the thought experiment consumption allocations cannot be adjusted, changing the timing of resolution of uncertainty only changes the degree to which consumption fluctuations are penalized, either by using σ if uncertainty is resolved early, or γ if uncertainty is resolved late. When $\sigma < \gamma$ ($\sigma > \gamma$) early (late) resolution entails a lower penalty and therefore it is preferred.

Empirical and experimental studies of financial risk have found that individuals are not indifferent to the timing of resolution of uncertainty as postulated by the EU model.[3] Although less is known for other kinds of lotteries, existing evidence suggests a preference for late resolution of uncertainty in the case of mortality risk. For example, field experiments find that many individuals choose not to learn their medical test results for various diseases. Given the clear instrumental value of information on health conditions, this evidence is at least suggestive of negative psychic benefits of early resolution (see Epstein *et al.*, 2013). Supporting evidence is found in the case of Huntington's disease, a fatal degenerative condition with onset around age 40. Using novel data for this disease, Oster *et al.* (2013) documents that, although genetic testing can perfectly predict it and carries little economic cost, testing is rare. In fact, untested individuals register behavior (fertility choices, retirement, etc.) that is identical to those not carrying the genetic expansion. Rare genetic testing in the case of this incurable disease suggests a preference for late resolution of uncertainty, or what is sometimes called "protective ignorance."

Many other studies on genetic testing for Huntington's disease also find that a sizable portion of the population at risk prefers not to know (Kessler, 1994; van der Steenstraten *et al.*, 1994; Tibben *et al.*, 1993; Yaniv *et al.*, 2004). Individuals cite as the major reasons to avoid being tested "fear of adverse emotional effects after an unfavorable diagnosis, such as deprivation of hope, life in the role of a patient, obsessive searching for symptoms and inability to support one's spouse" (Yaniv *et al.* 2004, p. 320). Wexler (1979) describes the results of 35 interviews with individuals at risk for the disease as follows: "All of the interviewers were painfully aware that the disease is terminal, but for them termination comes not at the moment of death but at the moment of diagnosis. Most fantasize the period following diagnosis to be a prolonged and unproductive wait on death row" (p. 199-220). Studies of HIV testing avoidance also find that many individuals exhibit some type of protective ignorance (Kellerman *et al.*, 2002; Day *et al.*, 2003; Weiser *et al.*, 2006). For example, Day *et al.* (2003, p. 665) conclude that the major barriers to voluntary counselling and testing were "fear of testing positive for HIV and the potential consequences, particularly stigmatization, disease and death."[4]

[3] For instance, Vissing-Jorgenson and Attanasio (2003) present empirical evidence to support a preference for early resolution of uncertainty. A limitation of this evidence is that it is hard to disentangle the attitude towards the psychic benefit of early resolution of consumption risk, from the instrumental or planning benefit of early resolution. However, available experimental evidence that isolates the psychic component supports a preference for early resolution of uncertainty in monetary lotteries (see Brown and Kim, 2014).

[4] If knowing the cause of death provides information about how painful death may be, then other considerations

The evidence on Huntington's disease supports a preference for late resolution of uncertainty. Individuals for whom termination comes not at the moment of death, but at the moment of diagnosis, could be rationalized from the perspective of preferences (2) as having underlying parameters $\sigma > 1 > \gamma$ and $W_{t+1} > \underline{W} = 0$. For these individuals, being diagnosed at time t with the disease is like learning that $\pi_t = 0$, so that death is a sure state next period. In this case, $E_t \widetilde{W}_{t+1} = \underline{W} = 0$, but also $W_t = 0$ if $\sigma > 1$. Intuitively, the low intertemporal elasticity parameter makes life miserable today if life will surely become miserable tomorrow. By avoiding testing, individuals can guarantee a positive utility today.

The health evidence suggests that individuals are not indifferent to the timing of resolution of mortality risk. The EZW representation provides superior flexibility relative to EU in that it captures the psychic effects of resolving this uncertainty at different points in time. Whether ultimately $\sigma \gtreqless \gamma$ is an empirical question.[5] Our quantitative analysis below sheds light on these parameters using information on the value of life both across countries, and across ages over the life cycle.

2.2 *Marginal valuation of survival*

A salient feature of the EU representation in (3) is that it is linear in survival probabilities. An implication of this linearity is that individuals attach the same value to a given change in survival $\Delta\pi_t$ regardless of whether the level of π_t is 5% or 95%. EZW preferences are more flexible because they allow for non-linearities in probabilities and therefore can recover a link between the level of survival and its marginal benefit. Whether survival exhibits increasing or diminishing returns depends on the degree to which consumption can be substituted across states and time. Specifically, the marginal rate of substitution between survival and consumption is given by

$$MRS_t = \frac{\partial V_t / \partial \pi_t}{\partial V_t / \partial z_t} = \beta \left(1-\gamma\right)^{-1} \left[\pi_t \left(V_{t+1}^{1-\gamma} - \underline{V}^{1-\gamma}\right) + \underline{V}^{1-\gamma}\right]^{\frac{\gamma-\sigma}{1-\gamma}} \frac{\left(V_{t+1}^{1-\gamma} - \underline{V}^{1-\gamma}\right)}{z_t^{-\sigma}}. \tag{6}$$

This expression shows that the marginal value of survival, or longevity, is positive if and only if $V_{t+1} > \underline{V}$. In addition,

$$\frac{\partial MRS_t}{\partial \pi_t} = \beta \frac{\gamma - \sigma}{\left(1-\gamma\right)^2} \left[\pi_t \left(V_{t+1}^{1-\gamma} - \underline{V}^{1-\gamma}\right) + \underline{V}^{1-\gamma}\right]^{\frac{\gamma-\sigma}{1-\gamma}-1} \frac{\left(V_{t+1}^{1-\gamma} - \underline{V}^{1-\gamma}\right)^2}{z_t^{-\sigma}},$$

arise. A painful death may effectively change the consumption allocation of an individual if he/she is unable to enjoy some of that consumption. From this perspective, one has to be careful when interpreting genetic test avoidance evidence. We thank an anonymous referee for pointing this out. Having said this, we think Huntington's disease constitutes a relatively clean case because fear of a painful death has not been cited as one of the major concerns for avoiding genetic testing.

[5] The increasing availability of genetic testing opens up the possibility of better documenting the role of preferences for late resolution of uncertainty. While identification is generally very difficult in other contexts, like financial risk, the availability of genetic testing provides an interesting opportunity in the case of mortality risk. Identification is still a key issue, but possible depending on the nature of the illness.

so that the marginal valuation of survival increases with π_t if $\gamma > \sigma$, decreases if $\sigma > \gamma$, and is constant if $\sigma = \gamma$. Intuitively, with a higher γ survival is increasingly more valuable because there is little substitution between the living and dead states, so that dying is particularly painful. On the other hand, the benefit of surviving diminishes when there is little intertemporal substitution (higher σ) because higher chances of survival increase future effective consumption, widening the intertemporal consumption gap.[6]

The link between the level of survival and its marginal valuation is potentially important. For instance, when discussing the value of life for the elderly, Becker *et al.* (2007) point at how "there may be inherent *non-linearity* in the valuation of life in the sense that the marginal valuation of an additional life year differs with the level of survival" (p. 6). A case they find particularly relevant is one in which "the more life one has the less one values an additional unit, similar to diminishing marginal utility in consuming other goods" (p. 7). From this perspective, a model in which the marginal valuation of extra life decreases with the chances of surviving would imply that the elderly value increases in survival more than what is predicted by a linear model. Since near the end of life the elderly have a lower survival probability, the EZW representation could explain why expenditures to prolong their life represent a particularly large fraction of health spending in the US. This issue is of increasing relevance for policy makers.[7]

Notice how parameters γ and σ determine both preferences for the timing of resolution of uncertainty, and the degree of diminishing or increasing returns to survival. For instance, $\sigma > \gamma$ implies both a preference for late resolution and decreasing marginal value of survival. The intuition for this result is tied to the fact that the effective time discount factor in EZW preferences is $\beta\pi_t[\pi_t + (1-\pi_t)\left(\underline{V}/V_{t+1}\right)^{1-\gamma}]^{(\gamma-\sigma)/(1-\gamma)}$.[8] If individuals are indifferent between early and late resolution, then the discount factor is proportional to π_t. Instead, if $\sigma > \gamma$ individuals are effectively more patient because in that case $\beta\pi_t[\pi_t+(1-\pi_t)\left(\underline{V}/V_{t+1}\right)^{1-\gamma}]^{(\gamma-\sigma)/(1-\gamma)} > \beta\pi_t$ as long as $V_{t+1} > \underline{V}$. A diminishing marginal value of survival is obtained in this case because the discount factor increases at a decreasing rate with π_t. The opposite holds when individuals prefer early resolution: for these relatively impatient agents the discount factor increases at an increasing rate with π_t. In the case of the elderly who have a lower survival rate π_t, an increase in π_t would raise the discount factor by relatively more if the agent is of the more patient type, one who prefers late resolution. This elderly individual would value the increase in survival relatively more than an otherwise similar

[6] Our analysis of the marginal valuation of survival focuses on the properties of preferences. We do not model here the "production" side of mortality risk, where competing risks may create a link between changes of survival and its level.

[7] We do not model endogenous health expenditures. A model of health spending with EZW preferences should also include technological progress in medicine which has been documented to explain the vast majority of health expenditure growth in the US (Chandra and Skinner, 2012).

[8] The marginal rate of substitution in consumption is

$$\begin{aligned}\frac{\partial z_t}{\partial z_{t+1}} &= \frac{\partial V_t/\partial z_{t+1}}{\partial V_t/\partial z_t} = \frac{\left(\partial V_t/\partial V_{t+1}\right)\left(\partial V_{t+1}/\partial z_{t+1}\right)}{\partial V_t/\partial z_t} \\ &= \beta\pi_t[\pi_t + (1-\pi_t)\left(\underline{V}/V_{t+1}\right)^{1-\gamma}]^{\frac{\gamma-\sigma}{1-\gamma}}\left(\frac{z_t}{z_{t+1}}\right)^{\sigma}\end{aligned}$$

Therefore, the implied time discount rate is $\beta\pi_t[\pi_t + (1-\pi_t)\left(\underline{V}/V_{t+1}\right)^{1-\gamma}]^{\frac{\gamma-\sigma}{1-\gamma}}$.

but younger individual who has a higher survival rate.

2.3 *Homotheticity with low intertemporal substitution*

In most quantitative macro models, including those at the intersection between health and macro, the elasticity of intertemporal substitution $1/\sigma$ is less than one (Murphy and Topel, 2006; Hall and Jones 2007; Jones and Klenow, 2011). In this class of EU models non-homotheticity arises naturally to guarantee that life has a positive bounded value. The non-homotheticity can take the form of a consumption floor or ceiling. In either case, non-homotheticity introduces both theoretical and quantitative issues. This section focuses on the theoretical aspects, while the quantitative aspects are discussed in the next section. An added advantage of EZW preferences is that they can preserve homotheticity even for the case $\sigma > 1$, without generating implausible predictions regarding the value of life.

Consider the EU framework as described in equation (5) when $z_t = z$ and $\pi_t = \pi$ for all t so that

$$V_t^{EU} = V_{t+1}^{EU} \equiv V(z, \underline{V}) = \left[\frac{z^{1-\sigma} + \beta\,(1-\pi)\,\underline{V}^{1-\sigma}}{1-\beta\pi}\right]^{\frac{1}{1-\sigma}}. \tag{7}$$

Let $\underline{z}$ denote the level of consumption for which $V = \underline{V}$ so that an individual with permanent consumption $\underline{z}$ would be indifferent between being alive or dead. Such level is given by $\underline{z} = (1-\beta)^{\frac{1}{1-\sigma}}\,\underline{V}$, the consumption equivalent of death. Individuals with consumption below $\underline{z}$ would prefer to die. Notice that it is not possible to avoid the non-homotheticity introduced by $\underline{z} > 0$ in EU models when $\sigma > 1$. This is because consumption in this case is essential in all states, so that having $\underline{z} = 0$, or $\underline{V} = 0$, makes lifetime utility zero $V_t = V(z, 0) = 0$ for all $z \geq 0$. In addition to not being a useful specification, in this case the price of survival would be infinite as can be seen from (6) when $\underline{z} = 0$ and $\gamma = \sigma$. Setting $\underline{z} = 0$ is not an issue when $0 < \sigma < 1$ because $V(z, 0) > 0$ for all $z > 0$, so consuming in all states is not essential.

In terms of the more familiar EU representation (3), the essentiality of consumption when $\sigma > 1$ is reflected in the fact that lifetime utility W_t^E is $-\infty$ when $\underline{z} = 0$. To see this, notice that using the transformation $W = V^{1-\sigma}/(1-\sigma)$, the case $\underline{V} \to 0$ corresponds to $\underline{W} \to -\infty$ when $\sigma > 1$, which results in $W_t^E = -\infty$.[9] To avoid this situation, $\underline{W} > -\infty$ is needed, or equivalently, $\underline{V} > 0$.[10] An alternative way to mechanically avoid the non-homotheticity when $\sigma > 1$ would be to set $\underline{W} = 0$ in (3), or equivalently $\underline{z} = \infty$, a consumption ceiling rather than a floor. In this case dying is always better than living regardless of the level of consumption. In fact, the price of survival in equation

[9] In the context of a fertility choice model, Doepke (2005, p. 340) uses this argument to restrict σ to be between zero and one.

[10] Readers may be more familiar with the idea of adding a constant to the utility flow rather than setting $\underline{W}$ to a negative value (e.g., Hall and Jones 2007). Both approaches are equivalent as shown by Rosen (1988). Subtracting $\underline{W}$ from both sides of equation (3), one can write

$$\widetilde{W}_t^{EU} = \frac{z_t^{1-\sigma}}{1-\sigma} - (1-\beta)\,\underline{W} + \beta\pi_t \widetilde{W}_{t+1}^{EU},$$

where $\widetilde{W}_t^{EU} = W_t^{EU} - \underline{W}$. In this version, the utility of dying is zero and the positive constant $-(1-\beta)\,\underline{W}$ is added to the utility flow.

(6) becomes negative when $\gamma = \sigma > 1$ and $\underline{V} = \infty$ (or $\underline{W} = 0$). This is not a very useful model of longevity because it portrays life as a bad rather than a good.

Non-homotheticities may introduce non-convexities in preferences, providing a welfare-improving role to life-or-death lotteries (Rosen, 1988). Figure 1 illustrates this issue for the EU model when $\sigma > 1$. Turning first to panel (a), if there are no lotteries and suicide is possible, then total utility will be non-convex: $V^{suicide}(z, \underline{V}) = \max\{V(z, \underline{V}), \underline{V}\}$. The solution in this case is $\underline{V}$ for $z < \underline{z}$ and $V(z, \underline{V})$ for $z \geq \underline{z}$, as seen in Figure 1a. The figure makes clear that non-homotheticity cannot be avoided: setting $\underline{V} = 0$ ($\underline{z} = 0$) would also imply $V(z, \underline{V}) = 0$ if $\sigma > 1$ (function $V(z, 0)$ coincides with the horizontal axis in Figure 1a).

The non-convexity of utility gives rise to potential gains through Russian-roulette type of lotteries as illustrated in Figure 1b. Individuals with consumption levels in the range $[0, z^*]$, with $z^* > \underline{z}$, would prefer to enter a life-or-death gamble that would pay z^* in the case of surviving and 0 if not. As noticed by Rosen (1988), convexification through lotteries guarantees that everyone's welfare is above $\underline{V}$ so that the economic value of a life is positive for all individuals ($z > 0$). Notice that in Figure 1b, function $V^{lottery}(z, \underline{V})$ corresponds to the convexified line in the range $[0, z^*]$, so that $V^{lottery}(0, \underline{V}) = \underline{V}$. In addition, $V^{lottery}(z, \underline{V}) = V(z, \underline{V})$ when $z > z^*$. Since under the gamble the survival probability is lower than π, then what effectively occurs is that "convexification is achieved by adopting modes of behavior that increase the risk sufficiently to enable survivors to attain consumption standard $[z^*]$" (Rosen, 1988 p. 289). This type of welfare-enhancing Russian-roulette lottery, however, is hardly observed in reality.[11]

Quantitative applications of EU to mortality questions generally abstract from gambles or suicide, leaving open the possibility that sufficiently poor individuals, with consumption below $\underline{z}$, do not value life nor extra years of life. As we argue in more detail in Sections 3 and 4, the EU literature faces a dilemma when calibrating $\underline{z}$, or equivalently $\underline{V}$. On the one hand, in the absence of lotteries, a small $\underline{z}$ is required to minimize the mass of individuals for whom life is a "bad." On the other hand, matching a plausible target for the value of (statistical) life requires a large $\underline{z}$ for the quantitative relevant case of $\sigma > 1$.[12]

An advantage of EZW preferences is that they can avoid non-homotheticity for any value of σ without implying that life is not valued. This is possible because EZW utility disentangles intertemporal substitution from risk aversion. This point can be seen from equation (4), which we rewrite here for convenience

$$V_t = \left[z_t^{1-\sigma} + \beta[\pi_t V_{t+1}^{1-\gamma} + (1-\pi_t)\,\underline{V}^{1-\gamma}]^{\frac{1-\sigma}{1-\gamma}}\right]^{\frac{1}{1-\sigma}}.$$

[11] We thank an anonymous referee for pointing this out. Although life-death lotteries are part of daily life, for example when crossing the street or flying, these are not welfare enhancing. In fact, individuals prefer to avoid this type of lottery or require compensation. For example, the main feature of labor contracts involving occupational risks is that individuals *are paid* to accept risky jobs. In contrast, the lotteries we refer to as uncommon are of the Russian–roulette type in which individuals are willing to *pay* to participate.

[12] Allowing for gambles could solve the problem of life being a "bad." However, not only these gambles are unrealistic, they would also imply that consumption levels below z^* should not be observed. As we show in Section 4, common calibrations of $\underline{z}$, and the fact that $z^* > \underline{z}$, imply that a significant fraction of the world population should not be alive because their observed consumption is below z^*.

As seen in this equation, $\underline{V}$ is raised to the power $1-\gamma$, not $1-\sigma$ as in EU. This implies that if $\sigma > 1$ consumption is essential at all *times*, but not in all *states*. What governs whether consumption is essential in all states is γ, the mortality aversion parameter. It is then possible to have $\sigma > 1$ as consistent with most evidence, while at the same time avoid non-convexities by setting $\underline{V} = 0$ ($\underline{z} = 0$). This requires the restriction $\gamma \in (0, 1)$, one that can be verified quantitatively. With $\underline{V} = 0$ all individuals with positive consumption value life. EZW utility can thus eliminate non-convexities and avoid situations when suicide is preferred, while at the same time being consistent with $\sigma > 1$. This is not possible in the standard EU model.

A possible, if troublesome, interpretation of negative values of life implied by existing quantitative exercises is that they represent cases in which life is not worth living due to extreme poverty, but suicide is costly.[13] Available evidence, however, provides no indication that even at low levels of consumption large numbers of individuals would prefer death to life. For instance, Banerjee and Duflo (2007) use household surveys from 13 developing countries to document how the extremely poor live. They find that "even the extremely poor do not seem to be as hungry for additional calories as one might expect" (p. 147). In fact, even if it appears the poor could spend more in food than they do, they allocate their spending in other valued nonfood items such as alcohol, tobacco and festivals. More importantly, "while the poor certainly feel poor, their levels of self-reported happiness or self-reported health levels are not particularly low" (p. 150). In a related study in a poor area of rural India, Banerjee *et al.* (2004) find no evidence of great dissatisfaction with life. Only 9% of those surveyed say their life makes them generally unhappy, a proportion very similar to what is found in the US. In general, life seems to be valued everywhere, even in extremely poor regions. What makes EZW utility more flexible than EU is the possibility of setting $\underline{z} = 0$ when $\sigma > 1$, so that non-convexities are eliminated and life is valued by all.

2.4 *Flexibility to match the empirical evidence on the value of life*

The marginal rate of substitution between survival and consumption, described in equation (6), measures the willingness of individuals to pay for additional life. This *price* is a key prediction of the model of particular importance for quantitative purposes. The corresponding price for the EU model, obtained by imposing $\sigma = \gamma$ in (6), is given by

$$MRS_t^{EU} = \beta (1-\sigma)^{-1} \frac{\left(\left(V_{t+1}^{EU} \right)^{1-\sigma} - \underline{V}^{1-\sigma} \right)}{z_t^{-\sigma}}. \tag{8}$$

Since the EU is a special case of EZW, the latter offers more flexibility when matching available evidence of the value of life, in particular, the value of statistical life (VSL) as we do in Sections 4 and 5. For given β and σ, the EU model can match a given target for the value of life by setting

[13] The macro-health literature does not explicitly include costs of suicide, nor the issues of non-convexities discussed here.

$\underline{V}$ properly, while the EZW model has two parameters, $\underline{V}$ and γ, to match the target.[14] This is a key advantage of EZW utility because it offers the possibility of choosing a small enough value of death $\underline{V}$ so that life is valued by all, allowing the mortality aversion parameter γ to be the main determinant of the value of life. As discussed above, it is even possible to choose $\underline{V} = 0$ so that life is valued by anyone with positive consumption. In contrast, under the EU model there is no guarantee that the value of $\underline{V}$ that matches a plausible target for the VSL is also consistent with all individuals valuing life extension. In fact, as we explain later, the EU model with $\sigma > 1$ faces an unavoidable tension because a low value of $\underline{V}$ is required to minimize the mass of individuals who do not value life, but as $\underline{V} \to 0$ (or $\underline{z} \to 0$) the value of life goes to infinity (see equation (8)).

Matching a target for the value of life using $\underline{V}$ as in the EU model, or γ as in with EZW utility has different economic implications. In particular, the first method introduces a non-homothetic element and income effects into the analysis, while the second uses an elasticity with no income effects implications per se. Finally, comparing equations (6) and (8) highlights another advantage of disentangling σ and γ. In principle, under EU σ also captures mortality aversion, playing a role in determining the VSL in (8). But in this case there is no guarantee that the same value of σ is consistent with the degree of intertemporal substitution estimated in the literature.

In sum, as a generalization of the EU model, EZW preferences make it possible to simultaneously match the imputed consumption upon death $\underline{z}$, the degree of intertemporal substitution in consumption $(1/\sigma)$, and the VSL (via γ). The fact that this can be achieved while maintaining recursivity, time consistency, and a parsimonious representation suitable for calibration makes EZW preferences appealing.

2.5 *Related approaches in the longevity literature*

Some attempts have been made to overcome the limitations of the time-and-state separable EU in the longevity literature. For instance, Bommier (2006) and Bommier and Villeneuve (2012) consider a modification of EU that relaxes time separability. This is achieved by introducing an endogenous discount factor that alters the computation of the VSL, and implies that preferences exhibit constant absolute risk aversion with respect to the length of life. Despite representing an interesting departure from time-separable EU, these endogenous discounting preferences remain within EU, so the underlying assumption of indifference to the timing of resolving death uncertainty still holds. In addition, the issues we discuss above regarding non-convexities and the need for gambles still apply.

In a recent working paper, Bommier (2014) explores the implications of longevity extension on aggregate wealth accumulation by using the risk-sensitive preferences of Hansen and Sargent (1995). As with EZW, these preferences also belong to the class introduced by Kreps and Porteus (1978). In addition to tractability and simplicity, another advantage of our EZW representation relative to Bommier's risk-sensitive utility is that we are able to guarantee preferences are convex by avoiding a minimum consumption level. In Bommier (2014), as in the rest of the longevity

[14] Parameters β and σ are typically obtained by matching targets related to the interest rate and the elasticity of intertemporal substitution.

literature, there is a non-convexity in utility when $\sigma > 1$ and the minimum consumption level is a key determinant of the value of life. In this respect, risk-sensitive preferences are subject to the dilemma discussed above, i.e., the minimum consumption should be small enough to avoid the need to introduce gambles, but large enough to match a plausible VSL. In contrast, in our model parameter γ, which governs aversion to mortality risk, is calibrated to match the VSL.

Bommier (2014) favors risk-sensitive preferences because under EZW utility there are instances in which parameter γ does not rank preferences in terms of risk. While this is generally true, it turns out that the EZW specification with $\underline{V} = 0$ is not subject to this limitation. Below we are able to prove this analytically for the case in which π_t and z_t are constant, and $\underline{V} = 0$ (see Section 5). In this case parameter γ does order preferences in terms of risk. This is not possible in the most common applications of EZW preferences in finance.

An alternative literature departs from EU by making different assumptions about how individuals perceive, or weight, survival probabilities. For instance, Bleichrodt and Eeckhoudt (2006) explore how the way individuals weight the objective survival probabilities affects their willingness to pay for reductions in health risk. The willingness to pay for reductions in health risks is larger when individuals underweight the probability of being in good health, or are pessimistic. This line of research is based on empirical studies showing that the probability weighting function is inverse S-shaped, overweighting small probabilities and underweighting large probabilities. The specific way this departs from EU is that the marginal utility of survival does depend on the level of survival. Although interesting, papers in this category do not address the issues of preference for timing of resolution, nor the issue of non-convexities discussed above.

In sum, to the best of our knowledge, ours is the first paper to use EZW preferences to study longevity issues. We next study a consumption and saving model with mortality risk and EZW utility.

3 A STATE NON-SEPARABLE UTILITY MODEL

This section derives the main theoretical results of the paper. The first part of the section solves a model in which individuals have EZW preferences, face age-dependent exogenous survival probabilities, and choose paths of consumption, leisure and saving to maximize lifetime utility. The main focus of the analysis is to derive the implications of the model for the marginal valuations of survival and the value of life during the life cycle. The second part of the section derives additional analytical results by considering the special case of perpetual youth.

For the analytical results of this section we set $\underline{V} = 0$ and focus on the role of γ, the coefficient of risk aversion, in determining the value of life. The formulation $\underline{V} = 0$ guarantees that all individuals with positive consumption prefer life over death, and life-or-death lotteries are not welfare improving. This case is also convenient for tractability and ease of comparison with related results in the literature, facilitating the exposition. It also enables us to clearly highlight the main differences between our approach and the standard EU approach used in the literature.[15] We

[15] Since EU is a special case of EZW preferences, EZW in this section refers to the case $\gamma \neq \sigma$ and $\underline{V} = 0$, while

consider the case $\underline{V} > 0$ in Section 5.1 where we document that the quantitative results obtained with $\underline{V} = 0$ are robust to plausible calibrations of $\underline{V}$.

Equation (4) with $\underline{V} = 0$ yields,

$$V_t = \left[z_t^{1-\sigma} + \beta \pi_t^{\frac{1-\sigma}{1-\gamma}} V_{t+1}^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \left[\sum_{s=t}^{\infty} \beta^{s-t} S(t,s)^{\frac{1-\sigma}{1-\gamma}} z_s^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \; \gamma \in (0,1), \; \sigma > 0, \tag{9}$$

where the sequential representation is obtained by recursive substitution of V_{t+s} into V_t, and $S(t,s)$ is the probability of surviving from period t to period s defined as $S(t,s) = \prod_{j=t}^{s-1} \pi_j$ for $s > t$ and $S(t,t) = 1$. When $\pi_j = \pi$ then $S(t,s) = \pi^{s-t}$. As we show in Section 4, calibrations of parameter γ under two very different scenarios, one cross-country and the other over the life cycle, indicate that $\gamma < 1$ so that the restriction $\gamma \in (0,1)$ is not binding. The same holds in Section 5 when we allow for $\underline{V} > 0$.

3.1 *Individual's problem*

Consider an individual of age t who survives to age $t+1$ with probability π_t, and holds initial assets a_t. The budget constraint of the individual at age t is given by

$$y_t + a_t = c_t + \frac{I(\pi_t)}{1+r} a_{t+1}, \tag{10}$$

where $y_t = w_t(1 - l_t) + b_t$ is total income, w_t is the wage rate, l_t is leisure, b_t non-wage income, c_t is consumption, $I(\pi_t)(1+r)^{-1}$ is a bond price, and r is the risk free interest rate. Function $I(\pi_t) = \delta\pi_t + 1 - \delta$ with $\delta \in [0,1]$ determines the degree of imperfections in annuity markets. The case $\delta = 1$ corresponds to perfect annuity markets, while $\delta = 0$ corresponds to no annuity markets. Individuals are assumed to retire at some exogenous age R. Let $z(c_t, l_t)$ be a composite good consisting of consumption and leisure. The individual's problem is described in recursive form as

$$V_t(a_t, \pi_t) = \max_{c_t, l_t, a_{t+1}} \left[H_t \cdot z(c_t, l_t)^{1-\sigma} + \beta \pi_t^{\frac{1-\sigma}{1-\gamma}} \left(V_{t+1}(a_{t+1}, \pi_{t+1}) \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{11}$$

subject to (10) and a natural borrowing limit. The formulation with $\gamma = \sigma$ resembles Murphy and Topel (2006). It allows for an exogenous health index $\{H_t\}_{t=0}^{\infty}$ affecting the quality of life, while $\{\pi_t\}_{t=0}^{\infty}$ determines the quantity of life.

EU refers to the case $\gamma = \sigma$ and $\underline{V} > 0$ as in Becker *et al.* (2005), Murphy and Topel (2006), Hall and Jones (2007), or Jones and Klenow (2011).

3.1.1 Optimality conditions

First-order conditions for assets and leisure are given respectively by[16]

$$H_t z_t^{-\sigma} \frac{\partial z(c_t, l_t)}{\partial c_t} \frac{I(\pi_t)}{1+r} = \beta \pi_t^{\frac{1-\sigma}{1-\gamma}} \left(V_{t+1}(a_{t+1}, \pi_{t+1})\right)^{-\sigma} \frac{\partial V_{t+1}(a_{t+1}, \pi_{t+1})}{\partial a_{t+1}}, \tag{12}$$

$$\frac{\partial z(c_t, l_t)/\partial l_t}{\partial z(c_t, l_t)/\partial c_t} = w_t \text{ for } t < R, \tag{13}$$

while the envelope condition reads

$$\frac{\partial V_t(a_t, \pi_t)}{\partial a_t} = V_t(a_t, \pi_t)^{\sigma} H_t z_t^{-\sigma} \frac{\partial z(c_t, l_t)}{\partial c_t}. \tag{14}$$

Equation (13) is the standard static labor-leisure choice condition. Using the envelope condition on the optimality condition for assets (12) we obtain the Euler equation for composite consumption z_t

$$\frac{z_{t+1}}{z_t} = \left[\beta (1+r) \frac{\pi_t^{\frac{1-\sigma}{1-\gamma}}}{I(\pi_t)} \frac{H_{t+1}}{H_t} \frac{\partial z(c_{t+1}, l_{t+1})/\partial c_{t+1}}{\partial z(c_t, l_t)/\partial c_t}\right]^{1/\sigma}, \tag{15}$$

which differs from the standard Euler equation in three ways. First, survival probability π_t matters in general for (composite) consumption growth unless $\gamma = \sigma$ and $\delta = 1$, the standard EU case with perfect annuity markets. In that case π_t does not enter into the Euler equation because both the marginal cost and the marginal benefit of saving are proportional to π_t. As seen in equation (12), with EZW utility the marginal cost of saving is still proportional to π_t if annuity markets are perfect, but the marginal benefit is proportional to $\pi_t^{(1-\sigma)/(1-\gamma)}$ via the discount factor. This is a key difference between the EZW and EU models, one that has implications for the life-cycle profiles of consumption and leisure. For instance, if $\sigma > \gamma$, then according to (15) composite consumption growth would tend to be higher than in the EU case because $\pi_t^{(1-\sigma)/(1-\gamma)} > \pi_t$. In this case the individual is effectively more patient, or prefers late resolution of uncertainty. Moreover, the effect of higher survival on consumption growth under EZW preferences can be negative, which is not possible under EU. This is the case for example, if annuity markets are perfect and $\sigma > \gamma$, or if annuity markets are absent and $\sigma > 1 > \gamma$. In both cases $\pi_t^{(1-\sigma)/(1-\gamma)}/I(\pi_t)$ decreases with π_t.

A second non-standard component in Euler equation (15) is the (gross) growth rate of the quality of life index H_{t+1}/H_t, which affects composite consumption growth. As in Murphy and Topel (2006), the main role of H_t is to help generate a realistic hump-shaped consumption profile in the absence of credit market imperfections. In particular, for retired individuals whose leisure is constant, as H_t declines when individuals age, consumption also declines and the consumption profile exhibits a hump (see Section 4.2 for details). Finally, the last term in the square brackets in (15) is the marginal rate of substitution between c_{t+1} and c_t which is potentially affected by leisure

[16] For simplicity we do not introduce new notation for optimal choices. Choice variables should be understood from now on to be the optimal ones.

choices.

3.1.2 The value of life

Consider next the willingness to pay for a longer life. In the model, the willingness of an individual of age t to pay for a procedure that increases the chances of survival by $\Delta\pi_t$ is given by

$$WTP_t(a_t, \pi_t) = \left|\frac{\partial a_t}{\partial \pi_t}\right| \Delta\pi_t = \frac{\partial V_t(a_t, \pi_t)/\partial \pi_t}{\partial V_t(a_t, \pi_t)/\partial a_t} \Delta\pi_t.$$

The envelope condition in (14) provides the expression for $\partial V_t(a_t, \pi_t)/\partial a_t$. Using equations (11) and (10), the marginal utility of survival is given by

$$\frac{\partial V_t(a_t, \pi_t)}{\partial \pi_t} = \frac{1}{1-\gamma} \beta \pi_t^{\frac{\gamma-\sigma}{1-\gamma}} \left(\frac{V_t}{V_{t+1}}\right)^{\sigma} V_{t+1} - V_t^{\sigma} H_t z_t^{-\sigma} \frac{\partial z_t}{\partial c_t} \frac{a_{t+1}}{1+r} \delta. \tag{16}$$

The first term on the right-hand side is positive: it multiples the utility of being alive at $t+1$, V_{t+1}, by a factor that captures the increase in the effective discount rate due to increased survival. The second term is negative: an increase in π_t carries a marginal cost because higher survival increases the price of bonds when annuity markets are present ($\delta > 0$). This term disappears from the valuation of life if annuity markets are absent.

The VSL, as defined in the literature, is the willingness to pay to save one life by a large pool of identical individuals. Since the overall willingness to pay by a population of size N is $N \times WTP_t(a_t, \pi_t)$, and the number of lives saved by the procedure is $N \times \Delta\pi$, then the VSL is given by

$$VSL_t = \frac{N \times WTP_t(a_t, \pi_t)}{N \times \Delta\pi} = \frac{\partial V_t(a_t, \pi_t)/\partial \pi_t}{\partial V_t(a_t, \pi_t)/\partial a_t}.$$

Thus, the VSL corresponds to the marginal rate of substitution between survival and assets. Using equations (16) and (14) we have that

$$VSL_t = \frac{1}{1-\gamma} \left[\beta \pi_t^{\frac{\gamma-\sigma}{1-\gamma}} \frac{V_{t+1}^{1-\sigma}}{H_t z_t^{-\sigma} \partial z_t/\partial c_t}\right] - \delta \frac{a_{t+1}}{1+r}. \tag{17}$$

An alternative way of writing the VSL formula, which allows direct comparison with existing results such as Murphy and Topel (2006), is given by

$$VSL_t = \frac{1}{1-\gamma} \left[\pi_t^{-1} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} I(t,s) \frac{z_s}{\partial z_s/\partial c_s}\right] - \delta \frac{a_{t+1}}{1+r}, \tag{18}$$

where $I(t,s) = \prod_{j=t}^{s-1} I(\pi_j)$ for $s > t$ and $I(t,t) = 1$.[17] This expression is similar to (17) but

[17] This formula is obtained using forward iteration on Euler equation (15), and writing equation (11) in sequential

it makes clear that term in brackets is the present value of *effective* consumption that would be lost in the event of death. In particular, term $z_s\left(\partial z_s/\partial c_s\right)^{-1}$ is composite consumption measured in consumption equivalent units. This "effective" consumption equals actual consumption in the absence of leisure. If the individual is neutral to mortality risk ($\gamma = 0$) then the value of a life is just the net present value of effective consumption lost in the event of death. Risk aversion ($\gamma > 0$) increases the value of a life above the consumption lost and in the limit, as $\gamma \to 1$, the value of life becomes infinite.

A further simplification can be obtained by substituting optimal savings in the expression. Forward iteration on the budget constraint (10) allows to write the VSL more compactly as

$$VSL_t = \pi_t^{-1} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} I\left(t,s\right) v_s, \tag{19}$$

where v_s is the value of a life-year as given by

$$v_s = \frac{1}{1-\gamma} \frac{z_s}{\partial z(c_s,l_s)/\partial c_s} + \left(y_s - c_s\right) \frac{\delta \pi_t}{\delta \pi_t + 1 - \delta}. \tag{20}$$

The VSL can thus be expressed as the present discounted sum of all the future values of a life-year, v_s. The value of a life-year in (20) has two components: *(i)* the effective consumption of that year $z_s\left(\partial z(c_s,l_s)/\partial c_s\right)^{-1}$ multiplied by a coefficient describing risk aversion, $1/(1-\gamma)$; and *(ii)* the contingent extra savings from annuity markets upon surviving. Equations (19) or (20) are analogous to the ones derived by Murphy and Topel (2006) for the EU case with perfect annuities. In fact, the only difference is in the expression of the value of a life-year. Theirs reads[18]

$$v_s^{EU} = \frac{1-\left(\underline{z}/z_s\right)^{1-\sigma}}{1-\sigma} \frac{z_s}{\partial z(c_s,l_s)/\partial c_s} + \left(y_s - c_s\right) \frac{\delta \pi_t}{\delta \pi_t + 1 - \delta}, \tag{21}$$

where $\underline{z}$ is the consumption equivalent of death. The comparison between the value of a life-year under EZW utility in (20) and EU in (21) indicates that the only difference between the two is the first term, the coefficient adjusting effective consumption. It is $1/\left(1-\gamma\right)$ in the EZW case, and $\left(1-\left(\underline{z}/z_s\right)^{1-\sigma}\right)/\left(1-\sigma\right)$ in the EU case. Given its importance in determining the value of life, we call this term the "gross mortality aversion premium", or simply GMAP, defined as

$$\theta\left(x,\gamma\right) \equiv \frac{1-x^{1-\gamma}}{1-\gamma}. \tag{22}$$

The GMAP is the factor by which effective consumption needs to be adjusted to properly reflect the value of a year of life in the absence of annuities. It is a premium if $\theta > 1$, or a discount if $\theta < 1$. If $\theta > 1$ the individual exhibits aversion to mortality risk in the sense that the imputed

form.

[18] See their equation 12, p. 880, which we extend to include the possibility of imperfect annuity markets.

compensation in the event of death is higher than the lost of effective consumption. In contrast, if $\theta < 1$ this imputed compensation is below the value of lost consumption. As we review below, and as first pointed out by Rosen (1981, p. 243), "almost all empirical estimates of willingness to pay find that it exceeds income" suggesting that $\theta > 1$ is the empirically relevant case. A feature of the case $\underline{z} > 0$ is that the GMAP depends on z_s and therefore varies during the life-cycle. One can define an average GMAP as

$$\overline{\theta}_t = \sum_{s=t}^{\infty} \theta\left(\underline{z}/z_s, \alpha\right) w_{t+s},$$

where $w_{t+s} = \left[(1+r)^{t-s} S(t,s) \frac{z_s}{\partial z(c_s,l_s)/\partial c_s}\right] \left[\sum_{s=t}^{\infty} (1+r)^{t-s} S(t,s) \frac{z_s}{\partial z(c_s,l_s)/\partial c_s}\right]^{-1}$. Coefficient $\overline{\theta}_t$ is the ratio of VSL to the present value of remaining effective consumption, both at age t, in the absence of annuities.

The GMAP for the EU case is obtained when $\gamma = \sigma$ and $x = \underline{z}/z_s$, while for (homothetic) EZW preferences the GMAP is constant over the life cycle so that $\overline{\theta}_t = \theta = 1/(1-\gamma)$.[19] The following proposition states some properties of the GMAP, which are particularly relevant for the EU case.

Proposition 1. *The gross mortality aversion premium $\theta(x, \gamma)$, or GMAP, has the following properties: (i) $\theta(0, \gamma) = \infty$ if $\gamma > 1$ and $\theta(0, \gamma) = 1/(1-\gamma) > 1$ if $v \in (0, 1)$; (ii) $\theta(1, \sigma) = 0$; (iii) $\theta(\underline{z}/z, \sigma) \gtreqless 1$ if $z \gtreqless \underline{z}\sigma^{\frac{1}{\sigma-1}}$; and (iv) $\theta_x(x, \sigma) < 0$.*

Part *(i)* of Proposition 1 shows that the GMAP, and therefore the VSL, can be infinite. In the EU model this occurs when $\underline{z} = 0$ and $\sigma > 1$. Since $\sigma > 1$ is the most common case in macro, then matching a finite VSL in the EU model requires $\underline{z} > 0$. Part *(ii)* of the proposition states that the GMAP could be as low as zero if consumption in the living and dead states is identical, i.e., $z = \underline{z}$. Part *(iii)* provides a lower bound on $z/\underline{z}$ in order for GMAP to be larger than 1. For example, if σ takes the standard value of 1.5 then the GMAP$\geq$1 if and only if $z \geq 2.25\underline{z}$. These three properties of the GMAP pose a dilemma for EU models such as Murphy and Topel (2006), Hall and Jones (2007), or Jones and Klenow (2011). On the one hand, a small $\underline{z}$ is required to minimize the mass of individuals for whom life is a bad (those with $z < \underline{z}$), or those with $\theta < 1$. But matching a plausible VSL requires a sufficiently large $\underline{z}$. Moreover, the non-convexity introduced by $\underline{z} > 0$ would create arbitrage opportunities for welfare-enhancing life-or-death gambles that are hardly observed in practice. Finally, part *(iv)* of the proposition implies that the GMAP is a positive function of effective consumption as long as $\underline{z} > 0$. Thus, individuals in poor countries, poor individuals in rich countries, as well as the youth and the elderly have a lower GMAP and value their life proportionally less. Section 4 below assesses the quantitative importance of these issues using calibrated models. These properties are not an issue for the EZW model because the case of $\sigma > 1$ does not impose any requirement on $\underline{z}$ being positive or particularly large.

In sum, with EZW utility: *(i)* life is a good for all individuals since $z \geq 0$ for everyone; *(ii)* the GMAP equals $1/(1-\gamma)$, which is always larger than one, constant during the life cycle, and the

[19] In Section 5.1 we show that in a perpetual youth model with non-homothetic EZW preferences ($\underline{z} \neq 0$), the GMAP is also described by equation (22) with $x = \underline{V}/V$.

same for the poor, the rich, the youth and the elderly; *(iii)* the value of life is finite and determined by γ, not by $\underline{z}$; and *(iv)* arbitrage opportunities for welfare-enhancing life-or-death gambles do not arise.

3.2 *Perpetual youth model*

A special case of the general framework in the previous section is the perpetual youth model: $\pi_t = \pi$, $H_t = 1$, $b_t = 0$, $w_t = w$, $y_t = y$, $z_t = c_t$, $l_t = 0$ for all t. Assume $a_0 = 0$ and let $\underline{z} = \underline{c}$ be the consumption equivalent of death. The EU version of this model is used by Becker *et al.* (2005) to compare well-being across countries. In Section 4.1 uses the EZW version for a similar purpose. This section uses the perpetual youth model to derive additional results and insights.

3.2.1 The value of life

Under the perpetual youth assumptions, $S(t,s) = \pi^{s-t}$ and $I(t,s) = I(\pi)^{s-t}$. In this case equation (15) simplifies to

$$c_s = c_t \left[\beta (1+r) \pi^{\frac{1-\sigma}{1-\gamma}} / I(\pi)\right]^{(s-t)/\sigma} \quad \text{for } s \geq t. \tag{23}$$

This equation together with budget constraint (10) and the initial asset condition $a_0 = 0$, can be used to solve for c_0 as[20]

$$c_0 = y \frac{1}{1+r-I(\pi)} \left(1 + r - I(\pi) \left(\beta (1+r) \pi^{\frac{1-\sigma}{1-\gamma}} / I(\pi)\right)^{1/\sigma}\right).$$

Using these two equations together with (19) and (20), the VSL in the perpetual youth model can be written as

$$VSL_t = \frac{\left((1-\gamma)^{-1} (I(\pi)/\pi) - \delta\right) c_{t+1}}{1 + r - I(\pi) \left[\beta (1+r) \pi^{\frac{1-\sigma}{1-\gamma}} / I(\pi)\right]^{1/\sigma}} + \frac{\delta y}{1+r-I(\pi)}. \tag{24}$$

Notice that the value of life is generally not time invariant because it is tied to consumption, which can increase or decrease over time depending on parameter values such as the interest rate. In addition, the effect of mortality on the value of life is rather complex. To gain some further intuition it is useful to consider the special case in which the interest rate is such that optimal consumption is constant, $c_t = y$, and the value of life is constant.

[20] This is provided that the term in parenthesis is positive. Boundedness conditions are assumed to hold throughout the paper.

3.2.2 Constant consumption in perpetual youth

Suppose $r = r(\pi) = I(\pi)\,\pi^{(\sigma-1)/(1-\gamma)}/\beta - 1$ so that, according to (23), optimal consumption is constant over time. In that case $c_t = y$ and (24) simplifies to

$$VSL(y,\pi) = \theta(0,\gamma)\frac{I(\pi)/\pi}{r(\pi)+1-I(\pi)}y, \tag{25}$$

while the corresponding expression for EU is given by

$$VSL^{EU}(y,\pi,\underline{c}) = \theta(\underline{c}/y,\sigma)\frac{I(\pi)/\pi}{r(\pi)+1-I(\pi)}y, \tag{26}$$

Notice that term $(I(\pi)/\pi)(r(\pi)+1-I(\pi))^{-1}y$ is the present value of income. With perfect annuities, equation (26) reduces to $VSL^{EU}(y,\pi,\underline{c}) = \theta(\underline{c}/y,\sigma)\times[y/(r(\pi)+1-\pi)]$, so the VSL is the present value of income adjusted by the GMAP. The resulting simplicity of the VSL makes transparent some of the earlier findings. Of special interest is the VSL-to-income ratio, which is defined as $\phi(y,\pi) \equiv VSL(y,\pi)/y$ and $\phi^{EU}(y,\pi,\underline{c}) \equiv VSL^{EU}(y,\pi,\underline{c})/y$ for EZW and EU respectively. The following proposition summarizes the main theoretical predictions of both models.

Proposition 2. *Consider the perpetual youth model with $\delta = 1$ and $1+r(\pi) = \pi^{(\sigma-\gamma)/(1-\gamma)}/\beta$. Under the EZW model: (i) $VSL(y,\pi) > 0$ for any $y > 0$, and $VSL(y,\pi) < \infty$ for any $\sigma \geq 0$; (ii) $\theta(0,\gamma) > 1$ for any y and σ; (iii) $\phi_y(y,\pi) = 0$ and; (iv) $\phi_\pi(y,\pi) > 0$ if $\sigma-\gamma < (1-\gamma)\,\beta\pi^{(1-\sigma)/(1-\gamma)}$ and $\phi_\pi(y,\pi) < 0$ otherwise.*
Under the EU model: (v) $VSL^{EU}(y,\pi,0) = \infty$ and $VSL^{EU}(y,\pi,y) = 0$ if $\sigma > 1$; (vi) $\theta(\underline{c}/y,\sigma) \geq 1$ if $y \geq \underline{c}\sigma^{\frac{1}{\sigma-1}}$ and $\theta(\underline{c}/y,\sigma) < 1$ otherwise; (vii) $\phi_y^{EU}(y,\pi,\underline{c}) > 0$ if $\underline{c} > 0$, and $\phi_y^{EU}(y,\pi,0) = 0$ if $\underline{c} = 0$; (viii) $\phi_\pi^{EU}(y,\pi,\underline{c}) > 0$.
Proof Most statements are immediate inspecting (25) and (26). Regarding part *(iv)* notice that $\phi(y,\pi) = (\theta\beta)/\left(\pi^{(\sigma-\gamma)/(1-\gamma)} - \beta\pi\right)$, so that

$$\phi_\pi(y,\pi) = \frac{\theta\beta}{\left(\pi^{(\sigma-\gamma)/(1-\gamma)} - \beta\pi\right)^2}\left[\frac{\gamma-\sigma}{1-\gamma}\pi^{(\sigma-1)/(1-\gamma)} + \beta\right],$$

which implies that if $\sigma - \gamma < (1-\gamma)\,\beta\pi^{(1-\sigma)/(1-\gamma)}$ then $\phi_\pi(y,\pi) > 0$. ||

Proposition 2 draws attention to two differences between EZW utility and EU. First, the behavior of the VSL-to-income ratio is very different. Ratio $\phi(y,\pi)$ is independent of income with EZW utility (and $\underline{V} = 0$), i.e., although the VSL is larger for higher income individuals, everyone values life in the same proportion to income. The same conclusion would hold in the EU case, but only when $\sigma < 1$ and $\underline{c} = 0$. Instead, if $\sigma > 1$ and therefore $\underline{c} > 0$, then the non-homotheticity implies that the VSL-to-income ratio is increasing in income under EU. In other words, a higher income individual would value life more than proportionally to his income. The value of life for the poorer individual is proportionally lower because his consumption is closer to the minimum $\underline{c}$.

Second, the VSL-to-income ratio is always increasing in the survival probability under EU. In this case the channel is purely discounting: since the discount factor in the EU model is $\beta\pi$, higher survival raises the VSL relative to income because the future utility is discounted at a lower rate. The VSL-to-income ratio may be increasing or decreasing in survival under the EZW model. For instance, part *(iv)* in Proposition 2 implies that $1 > \gamma \geq \sigma$ is a sufficient condition for this ratio to be increasing in survival. In this case individuals prefer an early resolution of uncertainty, and the elderly, who have lower survival rates, would value life relative to their incomes less than younger individuals. In contrast, if π is sufficiently close to one, $\sigma > 1$ is a sufficient condition for the VSL-to-income ratio to be decreasing in survival. In this case, since $\sigma > 1 > \gamma$, individuals prefer a late resolution of uncertainty and older individuals would have a higher VSL-to-income ratio than younger people.[21] The intuition for these results relates to the marginal utility of survival. When individuals prefer late resolution of uncertainty and remaining life is relatively scarce, as is the case for the elderly, then the marginal utility of survival is higher.

Proposition 2 suggests different qualitative implications for policy design depending on the formulation of preferences. Under EU, health interventions to increase survival probabilities for the younger and the richer would have a higher marginal value. Similar predictions hold for EZW utility if there is preference for early resolution of uncertainty, although the magnitudes would be different. For instance, a low-income individual who also has relatively low survival rate would have a lower VSL-to-income ratio than a high-income, high-survival individual. However, because of the non-homotheticity of utility, the EU model would predict a lower VSL-to-income ratio for the low-income, low-survival individual than the EZW model.

In contrast, if there is preference for late resolution of uncertainty, health interventions to increase the survival probability of older and poorer individuals would have a higher marginal value because, while the VSL-to-income ratio is independent of income under EZW, income and survival tend to be positively correlated. Individuals who are poor and have low life expectancy, or the elderly, would at the margin value increases in survival the most. In practice, the relevant scenario boils down to whether $\sigma \gtrless \gamma$. The following quantitative exercise illuminates this issue and illustrates the flexibility of the EZW framework.

4 QUANTITATIVE ANALYSIS

This section explores the quantitative implications of EZW preferences for the economics of longevity. We consider two different applications. First, we calibrate the perpetual youth version of the model

[21] It can be shown that this result does not depend on the assumption that the interest rate is $r = r(\pi) = I(\pi)\,\pi^{(\sigma-1)/(1-\gamma)}/\beta - 1$, case in which consumption is constant. The simplest way to see this is to consider the case of time-varying consumption with no annuities. When $\delta = 0$ equation (24) simplifies to:

$$VSL\left(c_{t+1}, \pi\right) = \theta(0,\gamma)\frac{c_{t+1}/\pi}{1 + r - \left[\beta\left(1+r\right)\pi^{\frac{1-\sigma}{1-\gamma}}\right]^{1/\sigma}},$$

and the VSL-to-consumption ratio, $\phi(c,\pi) = VSL\left(c,\pi\right)/c$, is decreasing in survival when $\sigma > 1 > \gamma$ (late resolution is preferred).

in Section 3.4, and use it to calculate full measures of per-capita income that include the gains in life expectancy between 1970 and 2005 for 144 countries. We compare our results to those obtained in Becker *et al.* (2005) for the EU model. In the second application, we calibrate our benchmark model with age-dependent survival and compute the implied VSL over the life cycle for the US. We compare our results with those obtained in Murphy and Topel (2006) for the EU model.

4.1 *Longevity across countries and time*

Except in cases of war or AIDS, significant gains in life expectancy occurred around the world between 1970 and 2005, with some poorer countries gaining as much as 26 years of life, and richer countries between 7 and 10 years.[22] In contrast, per-capita income differences have been quite persistent since 1970. With the exception of some growth miracles, income disparities remain quite stable. In fact, while there is substantial cross-sectional income inequality, with a number of countries below 10% of US per-capita income, there is less inequality in life spans, with no country below half of US life expectancy.[23]

How does world inequality look like if we take into account the joint evolution of per-capita income (quality of life) and longevity (quantity of life)? This question is analyzed in Becker *et al.* (2005) using an EU model of perpetual youth with perfect annuity markets, constant income and consumption, and a constant value of life. The spirit of their exercise is to think of the average individual in a country as receiving the per-capita income y of the country, and facing the constant survival probability π implied by the life expectancy T of the country, or $\pi = 1 - (1/T)$.

4.1.1 Calibration

We now calibrate the EU and EZW perpetual youth models of Section 3.4 following the methodology of Becker *et al.* (2005) as closely as possible, so that the only quantitative differences can be traced to the GMAP. Table 1 summarizes the calibration. For both models we exogenously set $\beta = 0.97$ and $\sigma = 0.8$ for all countries as in Becker *et al.* (2005). Countries differ in y and π in 1970 and 2005. We set the interest rate in each country to $1 + r(\pi) = \pi^{(\sigma-\gamma)/(1-\gamma)}/\beta$ so that consumption equals income y in every period.

The key calibration target, one that allows identifying $\underline{c}$ in the EU model and γ in the EZW model, is the VSL for the US. Estimations of the VSL are often based on wage differentials across occupations with different mortality risks, or from market prices for products that reduce fatal injuries. These approaches produce similar estimates of the VSL. To explain this concept, consider a worker who requires an annual premium of \$500 per year in order to accept an increase in the annual probability of accidental death of $1/10,000$. In a pool of 10,000 workers, one worker is expected to die and the aggregate compensation for such death is $VSL = \$500 \times 10,000 = \$5$ million. Actual estimates of the VSL in the US range between \$4 to \$9 million in 2004 dollars

[22] Data corresponds to life expectancy at birth from the World Development Indicators for a sample of 144 countries. We use life expectancy at birth, rather than at age 20, in order to compare our results with the literature. In the data, adult life expectancy is less dispersed than expected longevity at birth.

[23] Per capita income in PPP prices taken from the Penn World Tables Version 7.0.

for a 40 year old male (Viscusi, 1993; Viscusi and Aldy, 2003). These estimates have important policy implications and are used for policy evaluations. For instance, the Environmental Protection Agency has used \$6.3 million in cost-benefit analysis since 1993.

As Table 1 indicates, the value of $\underline{c}$ in the EU model that matches a VSL of \$2.9 million in the US is $\underline{c} = \$526$. This corresponds to the $\underline{c}$ obtained in Becker *et al.* (2005) but in 2005 prices. Becker *et al.* (2005) acknowledge that a VSL of \$2.9 million is low relative to the range reported in Viscusi and Aldy (2003), but we use the same value for comparison purposes. The value of γ in the EZW model that matches a VSL of \$2.9 million in the US is $\gamma = 0.594$. This calibration implies a preference for late resolution of uncertainty since $\sigma = 0.8 > \gamma = 0.59$. This value of γ should be regarded as an upper bound. If the model is instead calibrated to match a higher VSL for the US, as suggested in Viscusi and Aldy (2003), then γ is even lower. In sum, the data supports $\gamma < \sigma$. Last, recall that in this calibration $\underline{c} = 0$ in the EZW model. Section 5 checks the robustness of the results for $\underline{c} > 0$.

4.1.2 Predicted VSL-to-income ratios

Figure 2 portrays the cross-country VSL-to-income ratios in 2005 according to the EU and EZW models. It also shows alternative scenarios under EZW utility for two different plausible values of σ, namely $\sigma = 1.01$ and $\sigma = 1.25$. Turning first to our benchmark calibration, notice that under EU the ratio $\phi^{EU}(y, \pi, \underline{c})$ increases with income, as stated in Proposition 2. The increase in the VSL-to-income ratio is particularly sharp for income per-capita below \$10,000. This reflects both the non-homotheticity of the EU representation, and the fact that life expectancy for poorer countries rapidly increases with per-capita income. Recall that $\phi^{EU}(y, \pi, \underline{c})$ is increasing with π according to Proposition 2. Under the calibrated EU model six countries have a negative $\phi^{EU}(y, \pi, \underline{c})$ ratio, and therefore negative VSL, implying that the average individual in these countries would prefer to be dead rather than alive, and that convexification through life-or-death lotteries could be welfare improving.

As seen in Figure 2, the VSL-to-income ratio for EZW utility is always above the EU model, particularly for poorer countries. The ratio $\phi(y, \pi)$ is mostly flat for countries with income per-capita above \$5,000, around the sample median, and slightly increasing for incomes below this level. This reflects two facts: first, EZW utility is homothetic with $\underline{V} = 0$, so conditional on survival, ratio $\phi(y, \pi)$ is independent of income; and second, when γ is not very different from σ, the ratio $\phi(y, \pi)$ is increasing in π (Proposition 2). In sum, Figure 2 suggests the crucial role preferences play in studying longevity. The main insight of the figure is that with EZW utility, the VSL-to-income ratio in poorer countries, particularly those with income per-capita below \$10,000, is significantly larger than predicted by the commonly used EU model.

Regarding cross-country evidence on the value of life, Viscusi and Aldy (2003) report the VSL from 21 different studies around the world published since 1982 (see their Table 4, p. 27-28). Countries represented in these studies include richer nations such as Australia, Austria, Canada, Japan, and the United Kingdom, and developing economies such as Hong Kong, South Korea, Taiwan, and India. Although this international evidence tends to produce estimates of the VSL that are lower than in the US, the order of magnitude is similar despite the quite different labor market conditions across these countries. Both the calibrated EU and the EZW utility models predict that in rich countries the VSL is lower than in the US, but of similar order of magnitude. The main difference between the two models is in the predictions for poorer countries. Consider the case of India, the poorest country included in Viscusi and Aldy (2003). The most conservative estimate of the VSL in India they report is \$1 million.[24] In our calibration, the VSL in India is \$75,000 under EU and \$168,000 under EZW. Although well below the \$1 million estimate, the VSL with EZW is almost twice the one under EU. Two factors explain this difference. First, the VSL in India is lower under EU in part because the GMAP there is lower. Specifically, since in India $c = \$2,556$ in 2005, with $\underline{c} = \$526$ the GMAP $\theta(\underline{c}/c, \sigma) = 1.36$ under EU, while the GMAP $\theta(0, \gamma) = 2.46$ with EZW. Second, life expectancy at birth in India was 63 years, which implies that the effective discount factor under EU is $\beta\pi = 0.955$, while with EZW is $\beta\pi^{(1-\sigma)/(1-\gamma)} = 0.963$. Since according to the calibration late resolution of uncertainty is preferred, then individuals are slightly more patient under EZW, implying a larger VSL.

The benchmark calibration sets $\sigma = 0.8 < 1$ for comparison with Becker *et al.* (2005). However, in most quantitative macro studies $\sigma \geq 1$. Figure 2 displays the predictions of EZW utility for a value of σ close to one ($\sigma = 1.01$), and for $\sigma = 1.25$, the value used in Murphy and Topel (2006). Matching a VSL of \$2.9 million in the US with $\sigma = 1.01$ requires $\gamma = 0.501$. In this case, the ratio $\phi(y, \pi)$ is almost completely flat. In fact, equation (25) implies that when σ is close to one, then the effect of π on the VSL is negligible, i.e., regardless of life expectancy, the value of life relative to income is roughly the same across countries. At the other extreme, when $\sigma = 1.25 > 1$, then matching a VSL of \$2.9 million in the US requires $\gamma = 0.394$. As shown in Figure 2, in this case ratio $\phi(y, \pi)$ is decreasing, particularly for countries with income per-capita below \$5,000. The decreasing pattern is explained because in the calibration σ is sufficiently larger than γ, satisfying the corresponding condition in part *(iv)*, Proposition 2. The ratio $\phi(y, \pi)$ is particularly large for countries with short life expectancy. The average life expectancy in our sample is 67 years, with a minimum of 41 years. Almost all countries with life spans below 67 years also have income per-capita below \$5,000. When $\sigma > 1 > \gamma$, living as little as 41 years, or less than 67 years, makes life specially valuable. Such prediction cannot be generated by the EU model regardless of the values of σ and $\underline{c}$. Finally, going back to the case of India, the calibration of EZW utility with $\sigma = 1.01$ implies a VSL of \$175,443, which increases to \$184,750 for the case of $\sigma = 1.25$, around two and a half times the VSL with EU.

We now turn to calculate full measures of income that take into account both the quality and quantity of life.

[24] One may be worried that the estimates of the VSL are too high in developing countries, but as Viscusi and Aldy (2003) argue, this is likely not the case.

4.1.3 Full income and welfare

We now use the EU and EZW utility models to compute welfare measures over time and across countries. Consider a steady state along which the representative individual of a country consumes the per-capita income every year and faces survival π. According to (9), lifetime utility with EZW utility is given by

$$V(y,\pi) = \left[\frac{y^{1-\sigma}}{1-\beta\pi^{\frac{1-\sigma}{1-\gamma}}}\right]^{\frac{1}{1-\sigma}}, \tag{27}$$

while in the EU model, according to equation (3), it is given by

$$V^{EU}(y,\pi,\underline{c}) = \frac{1}{1-\beta\pi}\frac{y^{1-\sigma}-\underline{c}^{1-\sigma}}{1-\sigma}. \tag{28}$$

Let $V_0 \equiv V(y_0,\pi_0)$ be the welfare in a benchmark situation and $V_i \equiv V(y_i,\pi_i)$ the welfare in another situation i. For welfare measures across time, or growth calculations, the subscripts 0 and i refer to two different years for a given country, while for cross-country comparisons they refer to two different countries in a given year.

The typical measure of proportional welfare differences between these situations is the per-capita income ratio $R_i = y_i/y_0$. We now define a more comprehensive ratio of incomes that includes an imputed value for differences in life expectancy. We denote this ratio R_i^f where f stands for "full" income ratio which is defined implicitly by

$$V(R_i^f y_0,\pi_0) = V(y_i,\pi_i). \tag{29}$$

Thus R_i^f is the proportional change in y_0 required to equate welfare in both situations. Notice that $R_i^f = R_i$ if $\pi_0 = \pi_i$, and $R_i^f \lesseqgtr R_i$ if $\pi_i \lesseqgtr \pi_0$ and $y_i > \underline{c}$.

R_i^f for the EU and EZW utility cases can be easily obtained using (28) and (27). The solutions are given by

$$R_i^{f,EU} = \left[\frac{1-\beta\pi_0}{1-\beta\pi_i}\left(\frac{y_i}{y_0}\right)^{1-\sigma} + \left(1-\frac{1-\beta\pi_0}{1-\beta\pi_i}\right)\left(\frac{\underline{c}}{y_0}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{30}$$

and

$$R_i^f = \frac{y_i}{y_0}\left(\frac{1-\beta\pi_0^{\frac{1-\sigma}{1-\gamma}}}{1-\beta\pi_i^{\frac{1-\sigma}{1-\gamma}}}\right)^{\frac{1}{1-\sigma}}. \tag{31}$$

The solution for the EU case (30) is a CES function of the relative consumptions in the living and dead states with a weight, $(1-\beta\pi_0)\,/\,(1-\beta\pi_i)$, that measures the relative change in "effective mortality rates." The larger the reduction in mortality, the higher the weight assigned to the living state. Moreover, the lower the σ, the more substitution there is, and the larger the value imputed to mortality changes.

For welfare calculations across time, we use as situation 0 year 1970 and situation i year 2005 for each country. Figure 3 displays $R_i^{f,EU}/R_i$ and R_i^f/R_i as a function of the changes in life expectancy between 1970 and 2005. The vertical axis then measures the proportional change in welfare, in consumption equivalent units, due to changes in longevity. If all points clustered around the value of one on the vertical axis, then changes in longevity between 1970 and 2005 would not have altered welfare. Countries to the far right of the figure are those that substantially gained life expectancy, generally poorer countries. Those on the far left of the figure are also poorer countries, but they lost life expectancy mostly due to wars and AIDS. As richer countries had more modest gains in life expectancy, they are concentrated around zero on the horizontal axis. Figure 3 indicates that the EU and EZW utility models have similar predictions for richer countries: ratios $R_i^{f,EU}/R_i$ and R_i^f/R_i are closer to one for this set of countries. For the rest, however, the quantitative differences between the models are sizable. In particular, the EZW model penalizes welfare losses due to decreased life expectancy more than the EU model, while it values gains in life expectancy much more than the EU model. These results imply a decrease in world income inequality between 1970 and 2005, one that is specially pronounced according to the EZW model.

Table 2 reports the levels of full and per-capita incomes for countries at selected percentiles of the income distribution in 2005. In all reported countries, life expectancy increased between 1970 and 2005. Full incomes, which include these gains in life expectancy, are larger than per-capita incomes under both EU and EZW. The differences between full and per-capita incomes are larger for countries below the 50th percentile, as suggested by the annual growth rates. For instance, in Rwanda (10th percentile), the annual growth rate of per-capita income between 1970 to 2005 is 0.18%, while the corresponding growth rates of full income are 0.23% under EU and 0.50% under EZW. In Nigeria (25th percentile), the corresponding figures are 0.31%, 0.51% and 0.94%. Even at the 50th percentile (Guatemala), the respective annual growth rates are 0.94%, 1.53% and 1.86%. In contrast, for Hungary (75th percentile) and the US (98 percentile) the differences in annual growth rates are minimal. Table 2 confirms the finding that gains in life expectancy in poorer countries imply proportionally larger full income measures with EZW than with EU.

For welfare calculations across countries, we label the US as 0 and each of the other countries as i in equations (30) and (31). Figure 4 reports the results of cross-country welfare comparisons for 2005. Specifically, the figure plots $R_i^{f,EU}/R_i$ and R_i^f/R_i against life expectancy. If all points clustered around the value of one on the vertical axis, then differences in life expectancy would not matter for countries' welfare. As seen in Figure 4, the EZW model predicts that the low life expectancy in poor countries unambiguously reduces welfare by a large amount. For instance, according to the EZW model, a lifetime as short as 40 to 50 years reduces welfare by 40 to 50%. In contrast, under the EU model welfare is reduced by no more than 25%, or even increased by 20%, depending on the country's income level.

Table 3 confirms that when per-capita income is adjusted to reflect the gaps in life expectancy relative to the US, the cross-sectional dispersion of adjusted income is larger under EZW preferences. For instance, while per-capita income in Rwanda was \$839 in 2005, according to the EZW model the average individual there would be willing to accept an adjusted income of \$505 in exchange for having the same life expectancy of the US. Similarly, per-capita income in Nigeria is \$1,544 while adjusted income is \$902. As seen in Table 3, the differences between per-capita and adjusted incomes are smaller for countries beyond the 50th percentile. Last, while the standard deviation of the log of per-capita income in 2005 is 0.61, that of adjusted income with EZW preferences is 0.68. Cross-country welfare inequality is larger than income differences.

4.1.4 Wars and AIDS

The analysis above suggests that the EZW model significantly reassesses the economic consequences of major events affecting longevity such as the end of wars or the AIDS pandemic. Table 4 compares the predictions of the EU and EZW models for selected countries. We compute welfare across time using equations (30) and (31), selecting year 1990 as situation 0 and 2005 as situation i for each country. These dates are relevant to the AIDS pandemic. Countries in Table 4 are classified into two groups according to whether they gained or lost life expectancy between 1990 and 2005. Countries like Rwanda, Liberia and Niger gained 15.6, 8.5 and 8.1 years of life respectively. From the perspective of the EU model, these sizeable increases in life expectancy have only modest effects on welfare: the ratio of per-capita income is very similar to the full income ratio. For Niger, the ratios are equal, implying that a gain of 8.5 years of life is heavily discounted under EU. In contrast, a longer life span increases welfare with EZW preferences.

Consider now countries that lost years of life, mostly due to AIDS: Central Africa (3.2 years), South Africa (9.6), Botswana (13.3) and Zimbabwe (19.3). Both Central Africa and Zimbabwe experienced declines in years of life and per-capita income of 26 and 30% respectively. Table 4 indicates that the EZW model penalizes these shorter life spans much more than the EU model. For Zimbabwe, full income under EU dropped by 16%, less than the 30% drop in per-capita income, implying that 19.3 less years of life are welfare improving! In contrast, according to the EZW model Zimbabwe's full income sharply dropped, by 56%. Table 4 shows that the EZW model predicts substantial welfare costs of AIDS in Africa.

4.2 *The value of life over the life cycle*

In this section we explore the quantitative predictions of our benchmark model for the value of life over the life cycle. We now allow for age-dependent survival and leisure. Murphy and Topel (2006) computed the life-cycle profile of the VSL for the US using an EU model. In order to provide the cleanest comparison between the EU and EZW models, we replicate Murphy and Topel as much as possible, and calibrate the EZW using the same targets so that the only differences between the models can be traced to the GMAP.[25] Assume that composite consumption z_t is represented by

[25] A cross-country calibration with age-dependent survival could be performed, as life tables are available for a number of countries. However, since our purpose here is to illustrate the differences between the EU and EZW models, we restrict attention for the US economy. This comparison is more informative since Murphy and Topel (2006) have already provided life cycle estimates of the VSL for the US in the EU case.

the following CES function

$$z_t = z(c_t, l_t) = \left[(1-\rho)c_t^{\frac{\eta-1}{\eta}} + \rho l_t^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where ρ is the weight of leisure in composite consumption, and η is the elasticity of substitution between consumption c_t and leisure l_t. Although Murphy and Topel do not explicitly write the functional form of z_t in their paper, they indicate assuming a constant elasticity of substitution between consumption and leisure.

4.2.1 Calibration

Table 5 summarizes the calibration of the life-cycle model. We follow Murphy and Topel by exogenously setting $\sigma = 1.25$ and $\eta = 0.5$. The total number of available hours, work plus leisure, is set to $4,000$ a year. We use the US life tables from the 2000 National Vital Statistics Report to compute π_t, which is the probability of surviving to age $t+1$ conditional on having survived to age t. Hourly wages over the life cycle w_t are computed using data from the 2000 US Census. The sample is restricted to full-time working males between the ages of 20 and 64. The Census also provides information about yearly wage earnings and hours worked, both of which we use to inform our quantitative exercise. A fourth-degree polynomial in age is estimated to generate the wage profile. As in Murphy and Topel we assume benefits represent 29% of hourly wages, and life-contingent non-wage income b_t for ages 65 and higher is 50% of the wage earnings at age 64. Retirement is assumed to occur exogenously at age 65.

We calibrate parameters ρ and r, and the H_t profile so that both the EU and EZW models replicate the same consumption and leisure profiles in Murphy and Topel. This results in common ρ and r for both models, while H_t profiles differ. ρ is calibrated so that the ratio of consumption at age 50 to age 20 is 1.29, which yields $\rho = 0.029$. In addition, r is calibrated so that both consumption and income of the average working male equal \$52,493 at age 50, which results in $r = 3.38\%$.[26] Last, the health index H_t is calibrated so that annual consumption growth after age 50 is -2%.

In addition to the H_t profile, the main difference between the EU and the EZW models is the calibration of $\underline{z}$ in the former, and γ in the latter. In the EU model $\underline{z}$ is calibrated to match an average VSL between ages 25 and 55 of \$6.3 million. This is achieved by setting $\underline{z}$ so that $\underline{z}/z_{50} = 10\%$ at age 50 as in Murphy and Topel. This results in $\underline{z} = \$2,811$ and a value of a life-year at age 50 of $v_{50} = \$351,665$. The EZW model γ is calibrated to match an average VSL between ages 25 and 55 of \$6.3 million, the same VSL found in Murphy and Topel, which yields $\gamma = 0.684$. The implied value of a life-year at age 50 in the EZW model is $v_{50} = \$357,541$.

Figure 5 portrays the calibrated health index H_t for both the EU and EZW models, where the index at age 20 has been normalized to one, $H_{20} = 1$. H_t for the EU model replicates the one found in Murphy and Topel (2006, Figure 2.a, p. 887). We compute H_t for the EZW model as the

[26] As in Murphy and Topel (2006) we set β so that it satisfies $1 + r - 1/\beta = 2\%$.

residual from the Euler equation,

$$\frac{c_{t+1}}{c_t} = \left[\beta \pi_t^{\frac{\gamma-\sigma}{1-\gamma}} (1+r) \frac{H_{t+1}}{H_t} \left[\frac{(\rho/(1-\rho))^{\eta} w_{t+1}^{1-\eta} + 1}{(\rho/(1-\rho))^{\eta} w_t^{1-\eta} + 1} \right]^{\frac{1-\sigma\eta}{\eta-1}} \right]^{1/\sigma},$$

in order to replicate the same realistic hump shape for consumption as in Murphy and Topel.[27] By having both models replicate exactly the same consumption and leisure allocations, we ensure that the differences in the VSL reported below arise exclusively from differences in the GMAP.

Health indexes in Figure 5 are roughly constant until about age 35, when they start declining gradually up to age 70, and faster thereafter. The calibrated H_t declines faster in the EZW model. The reason is that under the calibrated EZW preferences, individuals are more patient, which would imply faster consumption growth than in the EU case. In order for both models to replicate the same consumption growth, H_t must be falling faster in the EZW case. We follow Murphy and Topel's calibration of H_t since our only purpose here is to compare the predictions of EU and EZW utility.[28]

Notice that the life-cycle calibration for the US and the cross-country calibration in Section 4.1 both imply γ is well below one, so that the restriction of $\gamma < 1$ is not binding. Recall also that for the perpetual youth model we chose a target of \$2.9 million for the VSL to keep the exercise comparable with Becker *et al.* (2005), while for the life-cycle model the target is an average VSL of \$6.3 million for ages 25 to 55 as in Murphy and Topel. Yet we continue to obtain $\gamma < 1$. In fact, had we calibrated the cross-country perpetual youth model to match a VSL of \$6.3 million in the US with $\sigma = 1.25$, we would have obtained $\gamma = 0.662$, very close to the calibration in this section.

4.2.2 The value of life

We now describe the life-cycle profile of the VSL implied by the calibrated EU and EZW models. Figure 6 portrays the VSL over the life cycle under EU and EZW utility. Profiles are drawn for an individual with average wage earnings, and for a low-income individual whose wage income over the life cycle is 50% of the average. The VSL in the figure corresponds to the value of "remaining life" at each age, as represented in equation (19), where the value of a life-year with EZW utility is given by (20) and that with EU by (21). Several patterns deserve comment. First, for the average-wage individual, the VSL is similar in both models, although the profiles cross twice over the life cycle. Specifically, the VSL is higher before age 40 and after age 80 under EZW preferences. The reason for this can be traced back to the GMAP in Proposition 1. Given that both models replicate the same allocations, the only difference in the values of a life-year according to equations (20) and (21) is the GMAP. While the GMAP is constant over the life cycle in the EZW case, it varies with age in the EU case depending on the ratio $\underline{z}/z_t$. As shown in Figure 7, the calibrated GMAP with EZW utility is constant and equal to $1/(1-\gamma) = 3.17$, regardless of the individual's age. In contrast,

[27] This Euler equation holds for working-age individuals between 20 and 64.

[28] An alternative way to generate hump-shaped consumption is to introduce credit constraints.

according to the EU model the GMAP for the individual with average wage earnings is lower than 3.17 before age 65; then it jumps to a higher value upon retirement, and then it falls below 3.17 around age 85. The jump in the GMAP at age 65 is due to the jump of composite consumption upon retirement induced by increased leisure. It is the relatively low GMAP at the beginning and at the end of life what lowers the VSL under EU for the average individual at those ages.

The pattern changes for the case of the low-income individual: the VSL is significantly higher under EZW preferences than under EU. In this case, as Figure 7 shows, the GMAP is always lower with EU than with EZW utility. Low-income individuals have a lower level of composite consumption z_t over their whole life cycle, which for given $\underline{z}$ results in a lower GMAP in the EU case. An alternative way of seeing the different implications of the model is to compute the ratio of the VSL under EZW utility relative to EU from Figure 6. For the low-income individual this ratio is always above one: around 1.3 at younger ages, and rising to 1.8 at older ages. In contrast, for the individual with average earnings, this ratio is slightly above one before age 40, below one between ages 40 and 80, and goes up to almost 1.2 after age 80. In a nutshell, the introduction of a minimum level of consumption $\underline{z}$ in the EU model has important implications for the age-profile of the VSL among individuals with different income levels.

5 FURTHER CONSIDERATIONS

5.1 *The non-homothetic case*

The results for the EZW model in Sections 3 and 4 are derived for the homothetic case, $\underline{V} = 0$. We now consider $\underline{V} > 0$ in the context of the perpetual youth model of Section 3.2. Our main qualitative and quantitative results are robust to plausible values of $\underline{V} > 0$, although differences may arise for the very poor individuals and countries. All calibrated values of the coefficient of risk aversion γ in various exercises remain significantly below existing estimates of σ, lending support to our approach of disentangling these parameters.

Assume perfect annuity markets. The Euler equation in this case reads

$$c_{t+1}^{\sigma} = \beta\,(1+r)\left[\pi + (1-\pi)\left(\underline{V}/V_{t+1}\right)^{1-\gamma}\right]^{\frac{\gamma-\sigma}{1-\gamma}} c_t^{\sigma},$$

which reduces to equation (23) when $\underline{V} = 0$. Equation (5) implies that $\underline{V} = \underline{c}/\left(1-\beta\right)^{1/(1-\sigma)}$ where $\underline{c}$ is the consumption equivalent of death.

Along the lines of Section 3.2.2, let $r(y, \pi, \underline{c})$ be the interest rate at which consumption is constant and equal to income y. In this case the VSL is given by

$$VSL\,(y, \pi, \underline{c}) = \theta(\underline{V}/V, \gamma)\frac{1}{1 + r(y, \pi, \underline{c}) - \pi - (1-\pi)\left(\underline{V}/V\right)^{1-\gamma}}y,$$

which reduces to (25) when $\underline{V} = 0$ and there are perfect annuity markets. The equation shows the role of the non-homotheticity in the EZW model. In particular, while for richer individuals or countries term $\underline{V}/V$ is negligible, it is quantitatively more important for determining the value of life for the poor. For the poor, the effect of having a low π and a low y on the VSL depends now on the extent of two opposing forces. Non-homotheticity implies that for countries in which y is close to the equivalent of death, $\underline{c}$, the willingness to substitute consumption for survival is lower than in the homothetic EZW case, decreasing the VSL. At the same time, if the marginal valuation of survival is decreasing in survival, then additional longevity would be highly valued in poorer countries with low π, increasing the VSL. The latter is the same channel present in the homothetic EZW case.

Figure 8 illustrates that our results continue to hold even when the EZW model is non-homothetic. This is particularly true for the case of poorer countries. The presence of $\underline{V} > 0$ in the EZW model requires adding a calibration target. To guarantee life extensions are valued in all countries, we calibrate $\underline{c}$ so that the average individual in the poorest country in the sample is indifferent between living and dying. This results in $\underline{c} = \$169$, which corresponds to the lowest per-capita income in the sample (Zimbabwe). For each value of σ considered in Figure 2, namely $\sigma = 0.8$, 1.01 and 1.25, we calibrate γ in the EZW model to match a VSL of \$2.9 million. This results in $\gamma = 0.697$, $\gamma = 0.568$ and $\gamma = 0.436$ respectively. This confirms the need to disentangle mortality aversion from intertemporal substitution. To calibrate the EU model, we follow the literature and choose $\underline{c}$ to match the same VSL for each of the σ values. This results in $\underline{c} = \$526$, $\underline{c} = \$2,380$ and $\underline{c} = \$4,750$ respectively. Finally, $\beta = 0.96$ as in Table 1. Figure 8 portrays the VSL in the EZW relative to the EU model for each σ.[29] The figure confirms that even when $\underline{V} > 0$ the VSL under EZW preferences is larger than (the absolute value) under EU, particularly for poorer countries.

There are two other points to notice in Figure 8. First, for $\sigma = 1.25$, a commonly used value in macro, the differences between the EZW and the EU models are the largest, in part because the EU model requires a larger $\underline{c}$ in order to match a given VSL, rendering the VSL negative for any country with per-capita income below \$4,750 (the sample median is \$5,367). This illustrates once more the quantitative pitfalls in the EU model. Second, notice that even though with $\underline{V} > 0$ the restriction $\gamma \in (0, 1)$ is not binding, since $\gamma < 1$ in all calibrations. The data clearly supports $\gamma < 1$.

5.2 *Ordering degrees of risk aversion*

Bommier (2014) argues that while the risk-sensitive preferences of Hansen and Sargent (1995) are well ordered in terms of risk aversion, this is not the case for EZW preferences. In particular, parameter γ may not order preferences in terms of degrees of risk aversion. While this is generally true, it turns out that our EZW specification with $\underline{V} = 0$ is not subject to this limitation. This

[29] The horizontal axis is truncated at a per-capita income of \$25,000 because for richer countries there are minimal differences across the models considered. In addition, to better view the pattern on the figure, the vertical axis is also truncated. Depending on the calibrated $\underline{c}$, a few countries have a VSL very close to zero in the EU model, implying high outlier ratios on the figure.

can be shown analytically in the simple perpetual youth model of Section 3.4.

Proposition 3. *Consider a perpetual youth model in which* $\pi_t = \pi$, $\delta = 1$, $H_t = 1$, $z_t = c_t$, $y_t = y$, $a_0 = 0$ *and* $r(\pi) = \pi^{(\sigma-\gamma)/(1-\gamma)}/\beta - 1$ *so that consumption is constant over time and equal to income* $c_t = y$. *Then under EZW utility with* $\underline{V} = 0$ *parameter* γ *orders preferences in terms of risk aversion.*

Proof Given $c_t = y$ for all t, forward iteration on lifetime utility (9) yields

$$V_0 = c\left(\sum_{s=0}^{\infty}(\beta\pi^{(1-\sigma)/(1-\gamma)})^s\right)^{\frac{1}{1-\sigma}} = c\left(1-\beta\pi^{\frac{1-\sigma}{1-\gamma}}\right)^{\frac{1}{\sigma-1}}. \tag{32}$$

If the individual is averse to mortality risk, then he must be better off by receiving the average consumption $E_0 c_t = \pi^t c$ at each t for certain, rather than facing the life-or-death lottery. Notice that in computing $E_0 c_t$ we are taking into account that the consumption equivalent upon dead is zero. Consider the lifetime utility of receiving $E_0 c_t$ in period t, alive or dead. Denote such utility $V(Ec)$. Then

$$V(Ec) = \left(\sum_{s=0}^{\infty}\beta^s (E_0 c_s)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} = c\left(\sum_{s=0}^{\infty}\left(\beta\pi^{1-\sigma}\right)^s\right)^{\frac{1}{1-\sigma}} = c\left(1-\beta\pi^{1-\sigma}\right)^{\frac{1}{\sigma-1}}, \tag{33}$$

so that the individual is mortality risk neutral if $V_0/V(Ec) = 1$, risk averse if $V_0/V(Ec) < 1$, and the lower this ratio is the larger the degree of risk aversion. Dividing (32) by (33) yields

$$\frac{V_0}{V(Ec)} = \left[\frac{1-\beta\pi^{\frac{1-\sigma}{1-\gamma}}}{1-\beta\pi^{1-\sigma}}\right]^{\frac{1}{\sigma-1}} \equiv h(\gamma,\sigma).$$

Given $\pi < 1$, the individual is risk neutral if $\gamma = 0$. Moreover, since

$$\frac{\partial h(\gamma,\sigma)}{\partial\gamma} = \left[\frac{1-\beta\pi^{\frac{1-\sigma}{1-\gamma}}}{1-\beta\pi^{1-\sigma}}\right]^{\frac{1}{\sigma-1}-1}\frac{\beta\pi^{\frac{1-\sigma}{1-\gamma}}}{1-\beta\pi^{1-\sigma}}\left(\frac{1}{(1-\gamma)^2}\ln\pi\right) < 0,$$

then mortality risk neutrality requires $\gamma = 0$, mortality risk aversion ($h(\gamma,\sigma) < 1$) requires $\gamma > 0$ and mortality risk loving requires $\gamma < 0$. Finally, the larger the γ the higher the degree of mortality risk aversion. Thus, parameter γ orders preferences in terms of risk aversion. ||

The result that EZW preferences with $\underline{V} = 0$ can properly rank life-or-death lotteries is novel. Most applications of EZW preferences are for financial markets, particularly asset pricing, where lotteries are of a different nature. The life-or-death lottery is special because when the imputed utility on the dead state is normalized to zero, only the utility in the living state enters explicitly in the representation. If $\underline{V} > 0$, then the EZW representation of the life-or-death lottery becomes similar to the ones of standard financial economics. In this case parameter γ does not always order preferences in terms of risk aversion. Since our EZW utility is positive, setting $\underline{V} = 0$ is a natural normalization as long as $0 < \gamma < 1$. Our calibrations above indicate that this restriction is not binding.

5.3 *Consumption versus mortality risk*

The calibration exercises across countries and over the life cycle imply a preference for late resolution of mortality uncertainty. In contrast, asset market evidence from the financial literature, such as the equity premium puzzle, suggests a preference for early resolution of *financial* uncertainty.[30] We can reconcile this seemingly conflicting evidence with an extension of our model that allows individuals to differentiate mortality risk from other types of consumption risks. For example, suppose this second type of risk is unemployment risk.[31] Consider two states $s = (e, u)$, where $s = e$ corresponds to employed and $s = u$ to unemployed. Let $V(s)$ be the utility of individual in state s,

$$V(s) = \left[(c(s))^{1-\sigma} + \beta \left[\pi \cdot E\left[V\left(s'\right)^{1-\eta} | s \right]^{\frac{1-\gamma}{1-\eta}} + (1-\pi) \cdot \underline{V}^{1-\gamma} \right]^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1}{1-\sigma}},$$

which is a generalized version of equation (4) to include both mortality and unemployment risk. In the equation above

$$E\left[V\left(s'\right)^{1-\eta} | s \right] = \lambda(s) V\left(e\right)^{1-\eta} + (1-\lambda(s)) V\left(u\right)^{1-\eta},$$

where $\lambda(s)$ is the conditional probability of being employed given status s the period before, and η is the coefficient of unemployment risk aversion. Normalizing $\underline{V} = 0$ yields

$$V(s) = \left[(c(s))^{1-\sigma} + \beta \pi^{\frac{1-\sigma}{1-\gamma}} E\left[V\left(s'\right)^{1-\eta} | s \right]^{\frac{1-\sigma}{1-\eta}} \right]^{\frac{1}{1-\sigma}},$$

which collapses to the standard EU when $\sigma = \gamma = \eta$. The representation above is flexible enough to accommodate a preference for the timing of uncertainty resolution for both mortality and unemployment risk. Specifically, it can simultaneously allow for: *(i)* $\sigma > \gamma$ or preference for late resolution of mortality risk, as consistent with our calibration above; and *(ii)* $\eta > \sigma$ to capture a preference for early resolution of unemployment (financial) risk. In sum, if $\eta > \sigma > 1 > \gamma > 0$ then we have a framework in which a preference for early resolution of unemployment risk coexists with a preference for late resolution of mortality risk. Our paper can be viewed as one in which there is no financial (unemployment) risk and therefore η plays no role.

[30] A summary of this literature is provided in Donaldson and Mehra (2008).

[31] We fully develop this set up with an application to welfare measures among OECD countries in Murtin *et al.* (2015).

5.4 *Alternative formulations of expected utility*

Our discussion of EU uses the popular constant relative risk aversion utility function. We now explore whether other momentary utility functions alter the characterization of EU presented above. Two alternative utility functions appear to change the prediction that the VSL is negative for poorer countries. First, suppose utility is a function of both market and non-market consumption. For instance, assume that utility is given by $(c+\omega)^{1-\sigma}/(1-\sigma)$, where $\omega > 0$ is the non-market consumption. In this case, the value of life could still be positive at low levels of consumption thanks to the presence of $\omega > 0$. Although this is certainly possible, we still find in various numerical exercises that ω would have to be sizable in order to obtain a reasonable calibration for the VSL, in the order to 65 to 95% of the total consumption $(c+\omega)$ in poorer countries. Even if one is willing to accept that non-market consumption is a large fraction of total consumption in poorer countries, this specification would still imply that the VSL-to-income ratio is increasing in income, and that individuals are indifferent to the timing of resolution of death uncertainty.

Another possible representation of EU that could in principle avoid negative VSL values for poorer countries is the following

$$u(c) = \begin{cases} ac \text{ for } c < \omega \\ a\omega + \frac{c^{1-\sigma}}{1-\sigma} - \frac{\omega^{1-\sigma}}{1-\sigma} \text{ for } c \geq \omega \end{cases}$$

where $a = \omega^{-\sigma}$. The formula above guarantees that the function is continuous and differentiable, and that marginal utility does not jump at threshold level ω. In addition, assume that $\omega \geq \underline{c}$ so that at least some, if not all, individuals below the threshold ω would prefer to be alive. Notice that it is still true that individuals with $c < \underline{c}$ would prefer to die. The linearity of utility at levels of consumption below ω implies that it is now possible to choose lower values of $\underline{c}$ without making the VSL very high. In other words, in the absence of concavity at low levels of consumption, the increase in marginal utility upon death is not as pronounced as when utility is concave. One would hope that by selecting an appropriate value for ω, one could obtain a calibration in which $\underline{c} < c$ for all countries so that the VSL is always positive, while keeping the VSL in the US within a reasonable range. This is indeed possible, but only with values in the upper range of existing estimates of the VSL in the US. Moreover, as in the case of non-market consumption, this threshold model still implies that the VSL-to-income ratio is increasing in income, and that individuals are indifferent to the timing of resolution of death uncertainty.

6 CONCLUDING COMMENTS

Our analysis makes the case for relaxing state separability of preferences when studying health and longevity issues. We discuss the case of EZW utility, a popular class of tractable functions that had found applications in the macro-finance literature, but has not been studied in the case of mortality. Expanding the set of state non-separable utility functions to analyze mortality risk is a promising research avenue.

Our quantitative exercises find that the parameters governing mortality aversion and intertemporal substitution are different. This appears to be a robust result, since it was obtained from calibrations that are distinct in many respects. According to the estimated parameters, individuals exhibit a preference for late resolution of death uncertainty and a decreasing marginal utility of survival. The added flexibility of EZW preferences is not limited to its better ability to match the evidence on the value of life. It also provides a different perspective on the behavioral aspects of preferences over life and death, and new insights that may change the policy implications of longevity models.

We document important quantitative differences between the EZW and EU models, particularly in terms of how the value of life changes with income, survival and age. Such differences matter when assessing the potential economic benefits of health interventions targeting specific groups such as the elderly or the poor. Moreover, our model can help rationalize the observed trends in health expenditures as population ages or the sizeable expenditures at the end of life. Finally, our model has implications for the analysis of health inequality within a country. The EZW model implies that health programs targeted at raising life expectancy of the poor may deliver significant welfare gains, much above those predicted under EU. We leave these questions for future work.

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TABLE 1

Parameters for cross-country exercise

Panel A – Exogenous parameters

	Parameter	Concept	Source	Parameter value
	σ	Inverse of intertemporal substitution	Becker, Phillipson and Soares (2005)	0.8
	β	Discount factor	Becker, Phillipson and Soares (2005)	0.97

Panel B – Calibrated parameters

	Parameter	Concept	Calibration target	Value target	Parameter value
EU model					
	$\underline{c}$	Minimum consumption	VSL in the US	$2.9 million	$526
EZW model					
	γ	Mortality risk aversion	VSL in the US	$2.9 million	0.594

TABLE 2

Full income in the time series: 1970 - 2005

Percentile income in 2005	Country	Life expectancy		Per-capita income	Full income in 2005		Annual growth income	Annual growth full income	
		1970	2005	2005	EU model	EZW model	1970 - 2005	EU model	EZW model
10th	Rwanda	44	48	839	853	939	0.18%	0.23%	0.50%
25th	Nigeria	40	47	1,544	1,652	1,919	0.31%	0.51%	0.94%
50th	Guatemala	52	70	5,629	6,892	7,712	0.94%	1.53%	1.86%
75th	Hungary	69	73	16,644	17,346	17,476	2.39%	2.51%	2.53%
98th	United States	71	78	42,535	46,314	46,314	2.11%	2.35%	2.35%
World									
	average	57	67	11,419	12,735	13,057	1.54%	1.82%	2.09%
	std. deviation	11	11	13,722	15,275	15,174	1.8%	1.90%	1.83%
	maximum	75	82	71,160	80,070	79,444	6.24%	6.58%	6.46%
	minimum	35	41	169	194	118	-4.74%	-4.87%	-4.10%

Notes: Life expectancy at birth is from the World Development Indicators. Per capita income is from the Penn World Tables, Version 7.0. Full income in 2005 includes gains in life expectancy between 1970 and 2005. EU refers to the case of the expected utility model, while EZW refers to the model with Epstein-Zin-Weil preferences.

TABLE 3

Adjusted income in a cross-section of countries: 2005

Percentile income in 2005	Country	Life expectancy	Per-capita income	Adjusted income	
				EU model	EZW model
10th	Rwanda	48	839	783	505
25th	Nigeria	47	1,544	1,316	902
50th	Guatemala	70	5,629	5,281	5,089
75th	Hungary	73	16,644	15,837	15,693
98th	United States	78	42,535	42,535	42,535
World					
	average	67	11,419	11,339	11,182
	std. deviation	11	13,722	14,042	14,131
	maximum	82	71,160	72,598	72,500
	minimum	41	169	219	83
	std. dev. log		0.61	0.61	0.68

Notes: Life expectancy at birth is from the World Development Indicators. Per capita income is from the Penn World Tables, Version 7.0. Adjusted income includes losses due to differences in life expectancy relative to the United States in 2005. EU refers to the case of the expected utility model, while EZW refers to the model with Epstein-Zin-Weil preferences.

TABLE 4

Gains from end of wars and losses from AIDS: 1990-2005

	Life expectancy 2005	Change in life expectancy 1990 to 2005	Per-capita income 2005 relative to 1990	Full income 2005 relative to 1990	
				EU model	EZW model
Gains in life expectancy					
Rwanda	48	15.6	1.08	1.18	1.93
Liberia	57	8.5	0.66	0.64	0.80
Niger	50	8.1	1.03	1.03	1.31
Losses in life expectancy					
Central Africa	46	-3.2	0.74	0.74	0.68
South Africa	52	-9.6	1.29	1.14	1.06
Botswana	51	-13.3	1.61	1.34	1.24
Zimbabwe	41	-19.3	0.70	0.84	0.44

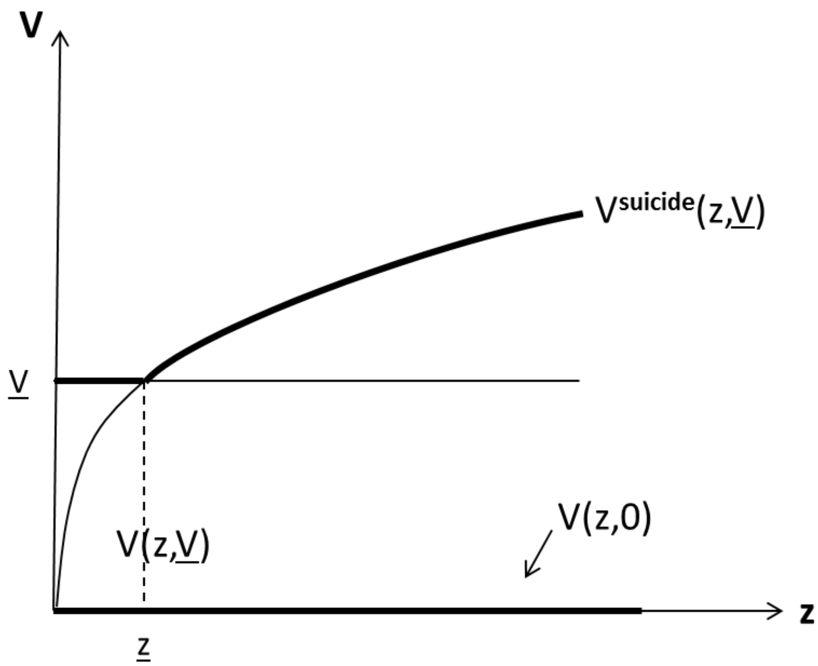
Notes: Life expectancy at birth is from the World Development Indicators. Per capita income is from the Penn World Tables, Version 7.0. Full income in 2005 includes the gains / losses in life expectancy between 1990 and 2005. EU refers to the case of the expected utility model, while EZW refers to the model with Epstein-Zin-Weil preferences.

TABLE 5

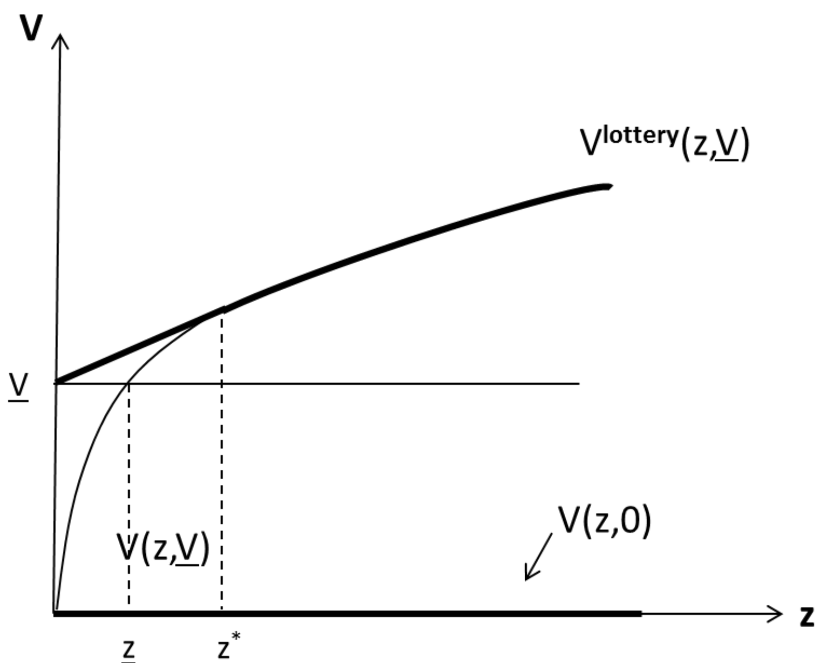
Parameters for life cycle quantitative exercise

Panel A – Exogenous parameters	Parameter	Concept	Source	Parameter value
	σ	Inverse of intertemporal substitution	Murphy and Topel (2006)	1.25
	η	Elasticity of substitution consumption and leisure	Murphy and Topel (2006)	0.5

Panel B – Calibrated parameters	Parameter	Concept	Calibration target	Value target	Parameter value
EU and EZW models					
	ρ	Share of leisure in composite consumption	Consumption at age 50 relative to age 20	1.29	0.029
	r	Interest rate	Consumption at age 50	$52,493	3.38%
EU model					
	$H(t)$	Life cycle health index	Annual consumption growth after age 50	-2.0%	Figure 5
	$\underline{z}$	Minimum composite consumption	Average VSL in the US between ages 25 and 55	$6.3 million	$2,811
EZW model					
	$H(t)$	Life cycle health index	Annual consumption growth after age 50	-2.0%	Figure 5
	γ	Mortality aversion	Average VSL in the US between ages 25 and 55	$6.3 million	0.684



a. Without lotteries



b. With lotteries

FIGURE 1
Value function with and without lotteries ($\sigma > 1$)

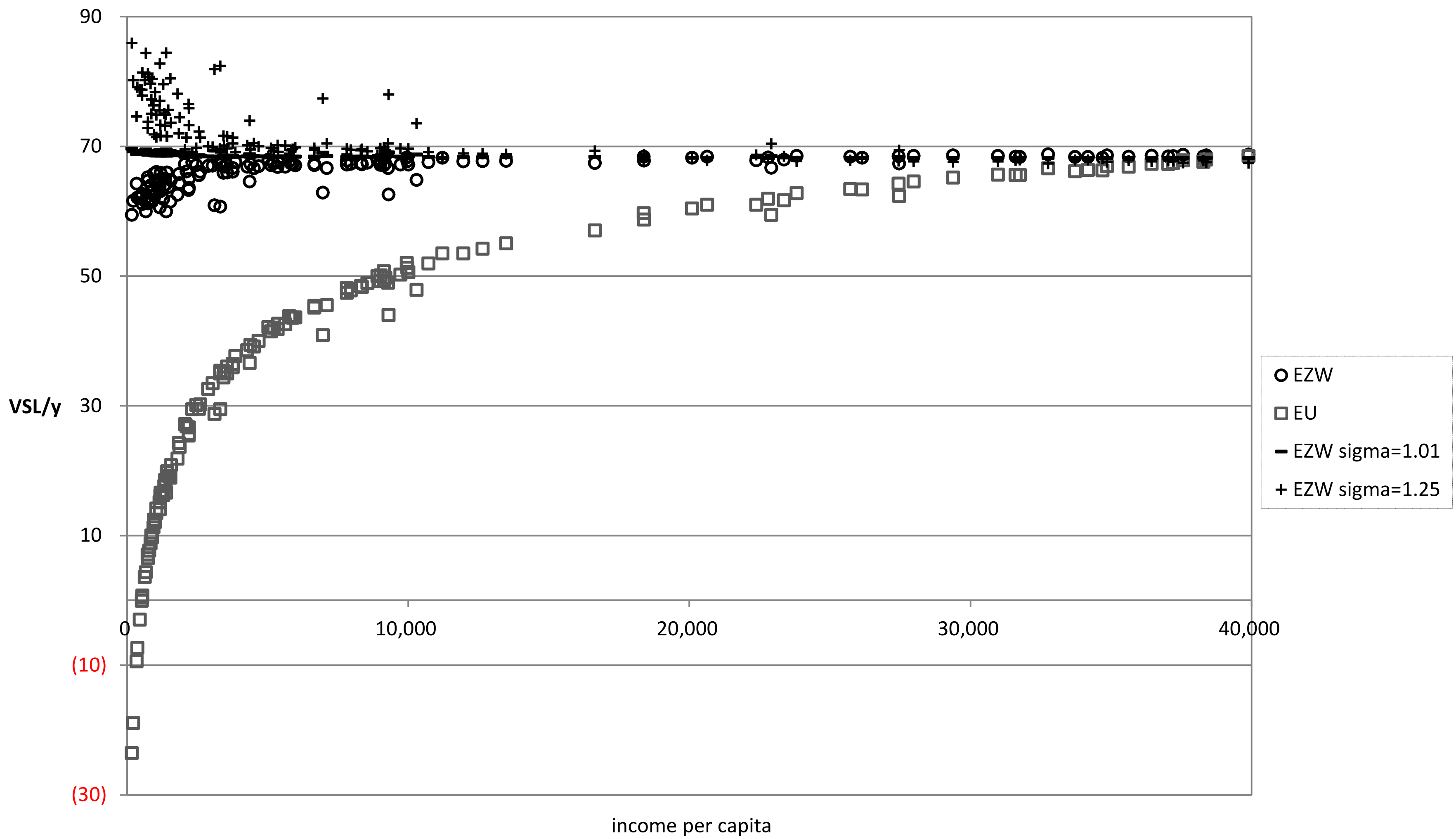


FIGURE 2
VSL-to-income ratio and income per capita across countries - 2005

Notes: VSL (value of statistical life) is computed as the marginal rate of substitution between assets and survival according to both the EZW and the expected utility (EU) models. The VSL corresponds to the overall willingness to pay to save a life. Per capita income in 2005 is from the Penn World Tables 7.0.

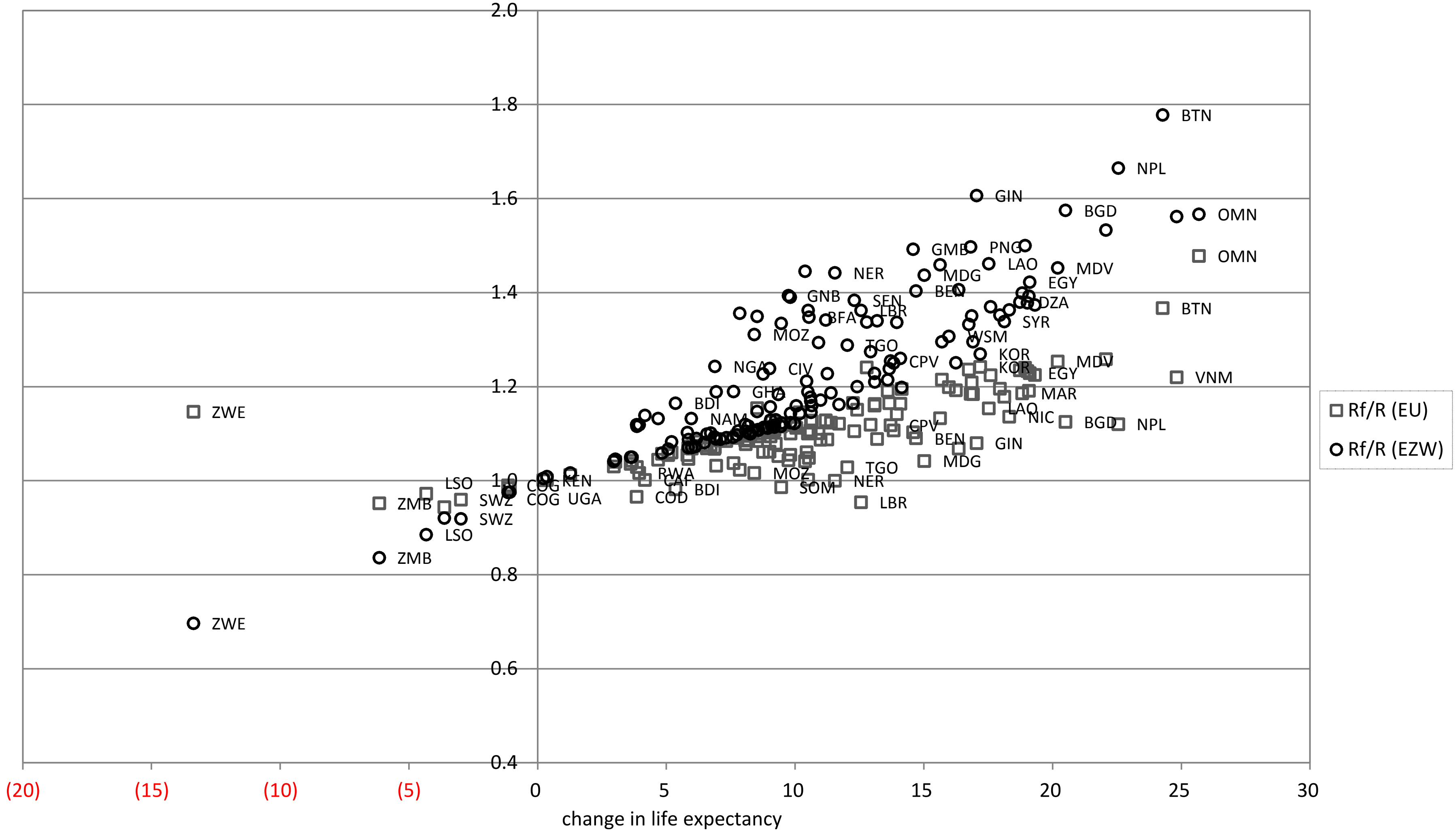


FIGURE 3

Relative contribution of longevity changes to welfare: 1970-2005

Notes: R is per-capita income in 2005 relative to 1970. Rf is full income in 2005 relative to 1970. Full income includes the value of gains in longevity according to the EZW and the expected utility (EU) models. When Rf/R equals one, there is no difference between per-capita and full income.

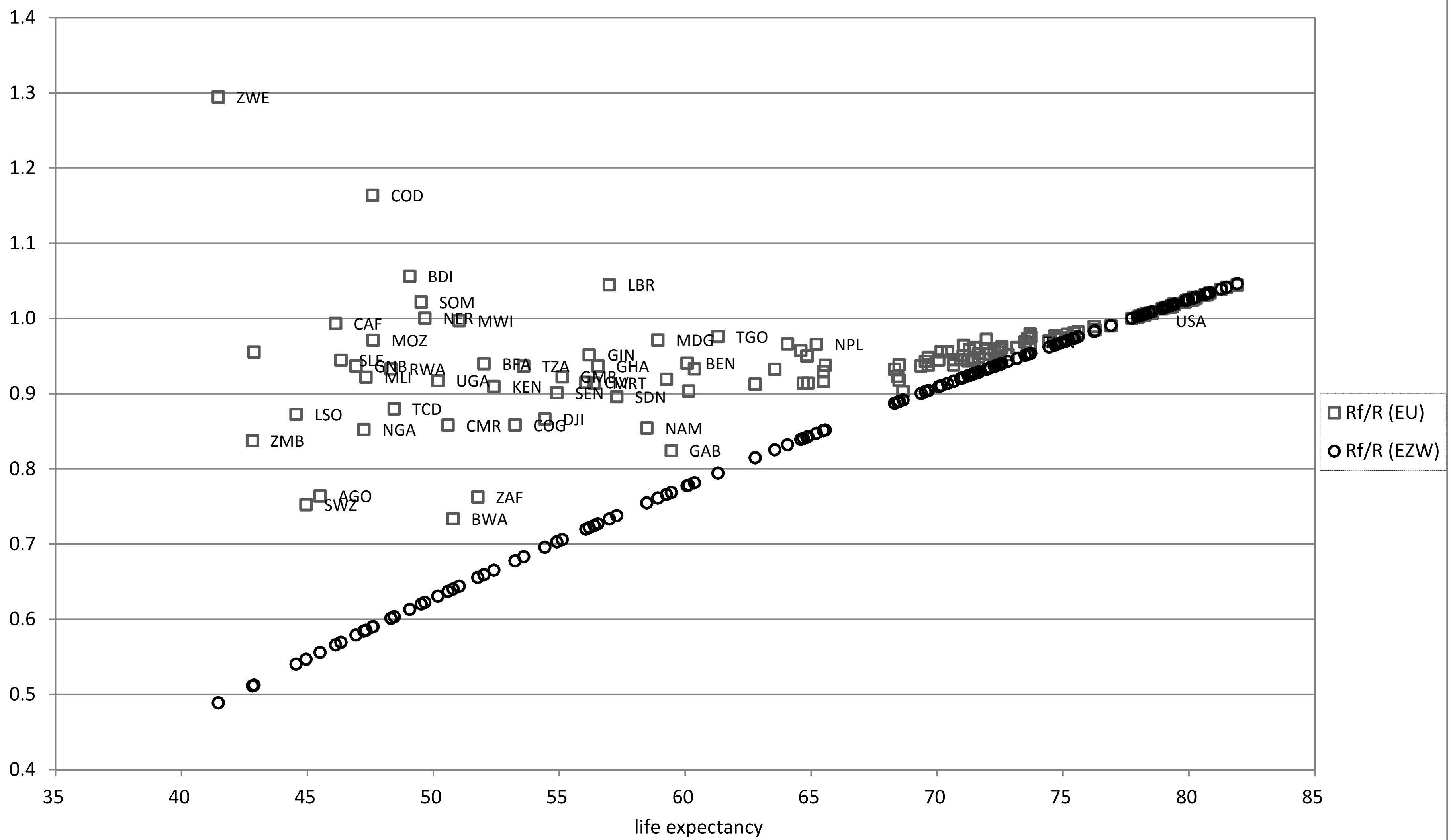


FIGURE 4

Relative contribution of longevity to welfare - Cross-section 2005

Notes: R is per-capita income relative to the US in 2005. Rf refers to adjusted income relative to the US. Adjusted income reflects losses due to differences in life expectancy relative to the US. When Rf/R equals one, there is no difference between per-capita and adjusted income.

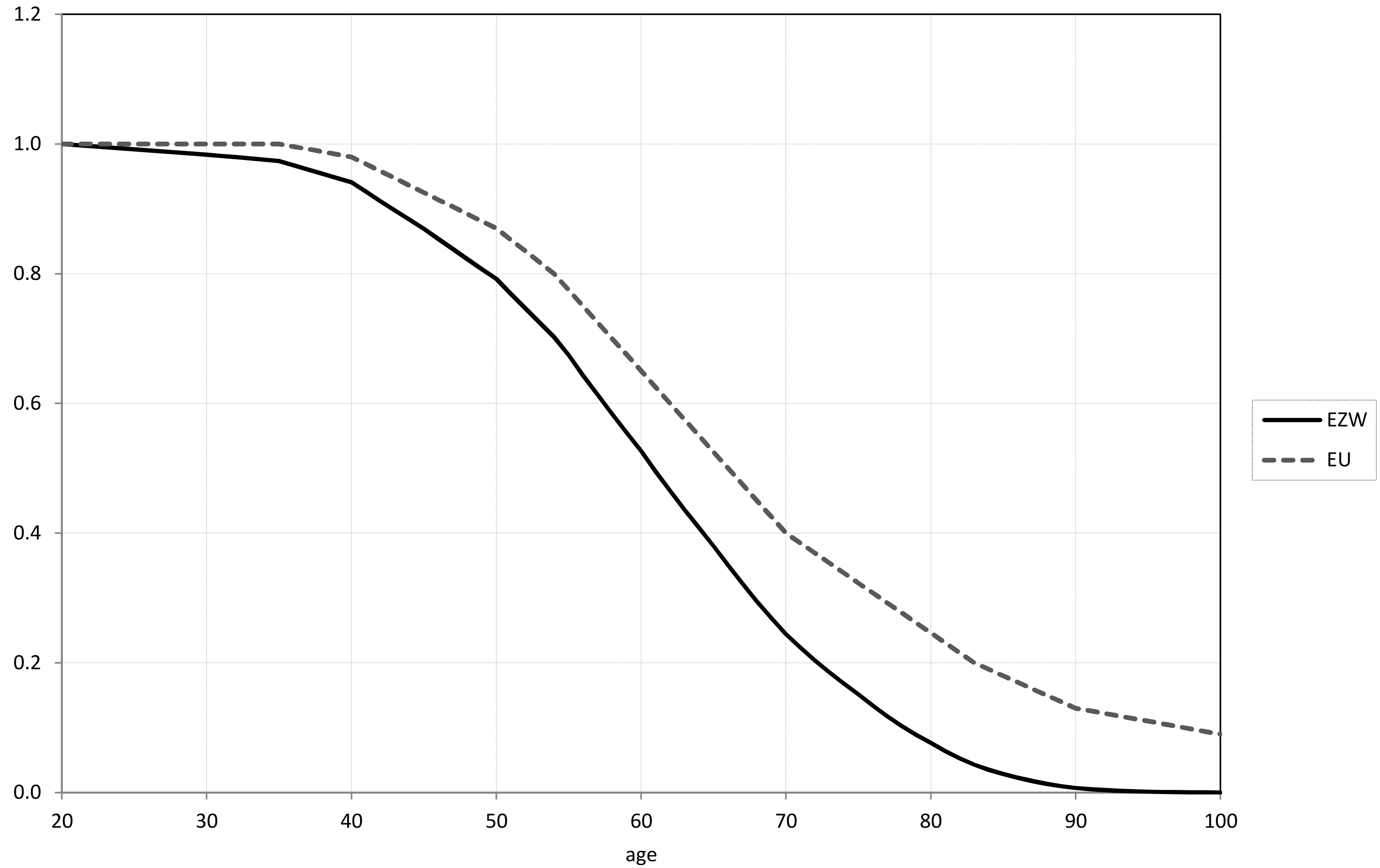


FIGURE 5
Health over the life cycle in the United States - 2000

Notes: The health index *H(t)* over the life cycle is computed as the residual from the Euler equation necessary to match an observed consumption profile. The health index is normalized to one at age 20.

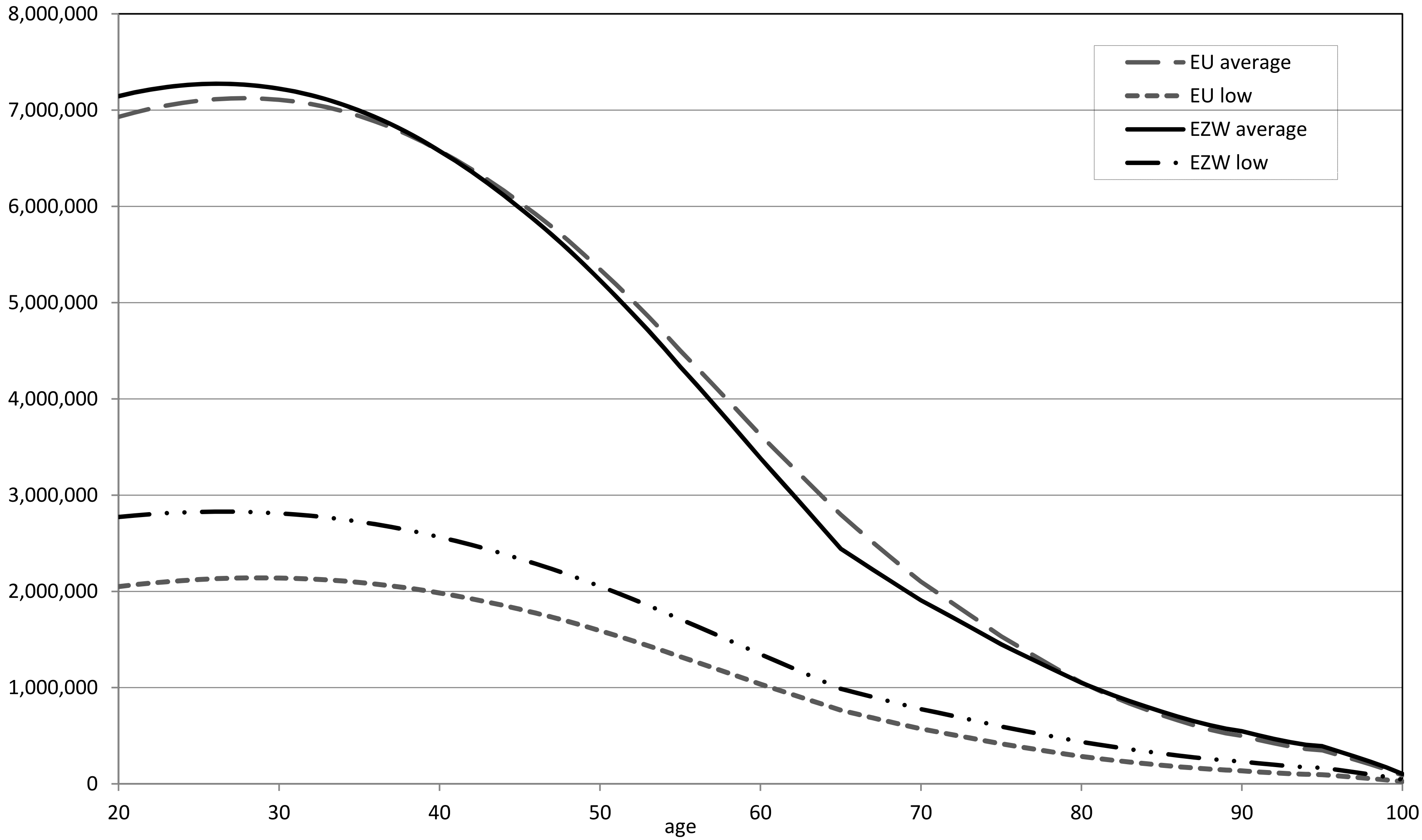


FIGURE 6
Value of statistical life over the life cycle - United States - 2000

Notes: The value of statistical life is computed as the marginal rate of substitution between assets and survival according to both the EZW and the expected utility (EU) models. The average individual has the mean wage earnings profile over the life cycle, while the low-income individual has 50% of the mean earnings.

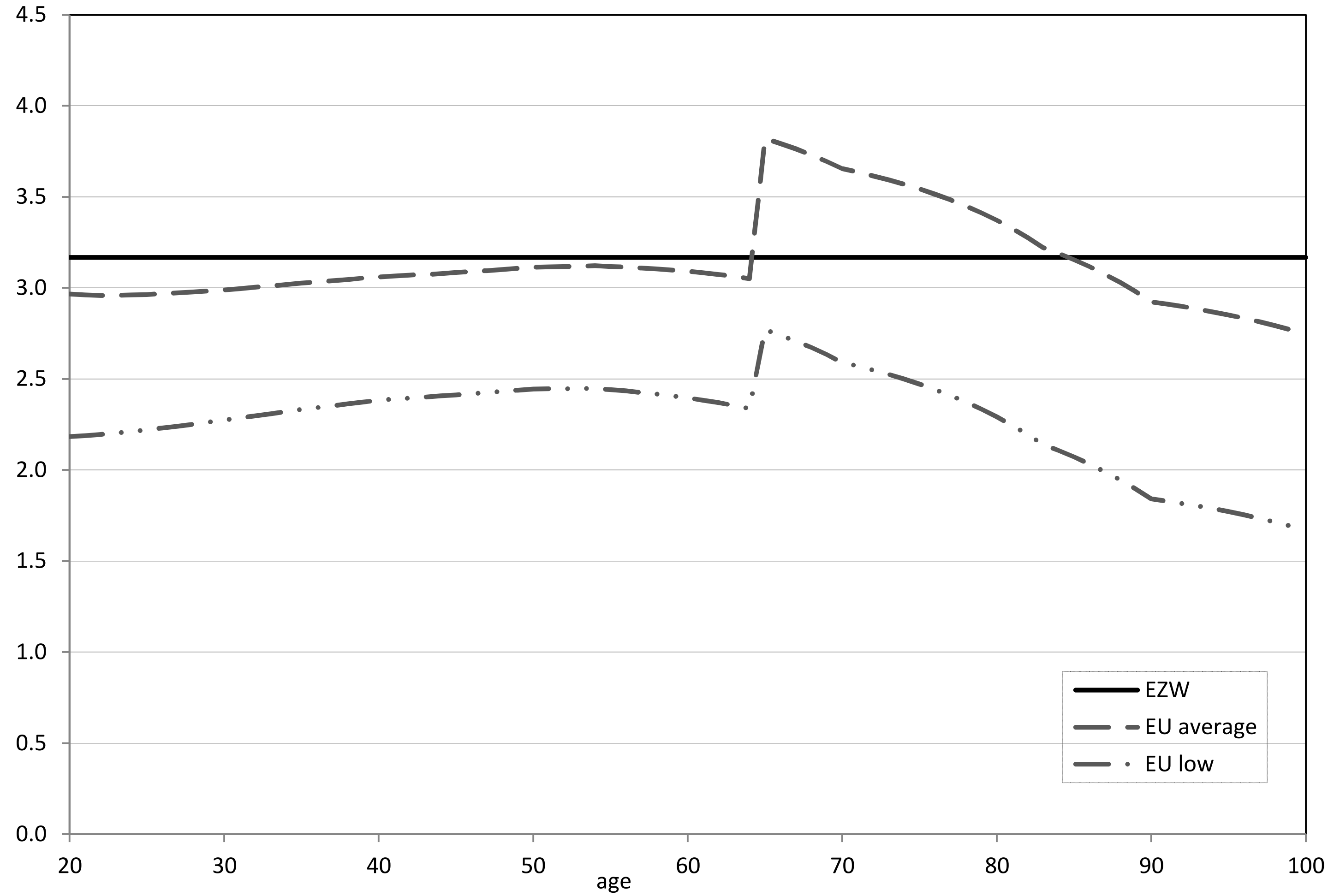


FIGURE 7
Gross mortality aversion premium - United States - 2000

Notes: The gross mortality aversion premium (GMAP) is the ratio of the value of a life year over effective yearly consumption. The average individual has the mean wage earnings profile over the life cycle, while the low-income individual has 50% of mean earnings.

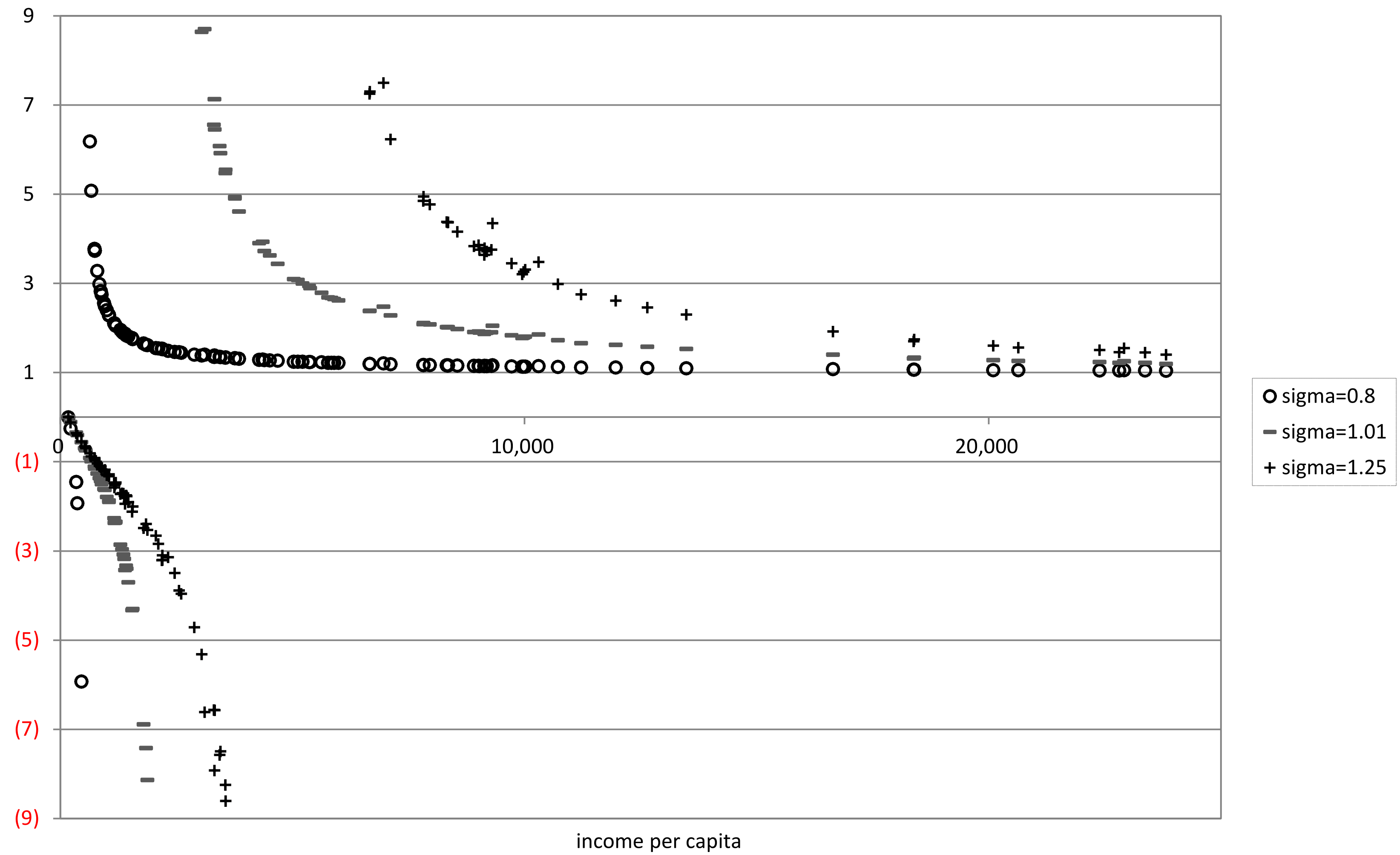


FIGURE 8
VSL in the non-homothetic EZW relative to the EU model - 2005

Notes: Same as Figure 2. Under the non-homothetic EZW model imputed utility upon death is positive.