Agriculture and aggregation

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Abstract

We derive a multiplicative representation of the aggregate production function in a two-sector economy. This representation includes a "diversification component" that is a function of factor allocation across capital-intensive nonagriculture, and land-intensive agriculture. Since this component is large in very poor countries, their productivity levels are lower than the implied by Solow residuals.
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1 Introduction

It is a well-known fact that in less developed countries agriculture accounts for a larger share of output and employment than in richer economies. In addition, agriculture is more intensive in the use of land than nonagriculture. These facts suggest the existence of differences in sectoral composition across countries, and in production technologies across sectors. How do these sectoral differences affect the aggregate representation of the production function? And, how do they affect our understanding of the sources of cross-country income differences?

This paper studies the sources of cross-country income differences in a world composed by a large number of small open economies, each of which produces an agricultural and a nonagricultural good. We characterize the aggregate production function for the general case in which sectors differ in their land, capital, and labor intensities. We assess the extent to which the Solow residuals obtained from the standard one-sector model properly characterize the underlying technological differences

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across countries. Finally, we use the derived aggregate production function to perform a variance decomposition of output per worker.

The analysis yields three main results. First, when sectoral technologies are Cobb-Douglas, we can derive a multiplicative representation of the aggregate production function that is suitable for variance decomposition. This representation includes a "diversification component" that captures the efficiency gains of operating both sectors, and is a function of factor allocation across capital-intensive nonagriculture, and land-intensive agriculture. Second, the diversification component is negatively correlated with per capita income. This negative correlation is mainly driven by the very poor countries in the sample. We find that the technological gap between, for instance, Uganda and US in nonagriculture is 15% larger than what is implied by Solow residuals. Finally, we find that although productivity in very poor countries is lower than the one implied by Solow residuals, development accounting exercises for the cross-section of countries using our methodology are only marginally different from those using one-sector models.

Our paper is related to recent work on agriculture and development (Chanda and Dalgaard, 2008; Gollin et al., 2004; Restuccia et al., 2008; Temple and Woessman, 2006; and Vollrath, 2008), but it is closest to Caselli (2005). A distinguishing feature of our work is that we explicitly derive a multiplicative representation of the aggregate production function for a two-sector model, and characterize the Solow residual as a function of factor allocation across sectors. We develop multiplicative results that allow the application of variance decomposition techniques from one-sector models into two-sector models.

The remainder of the paper is organized as follows. Section 2 derives the aggregate production and provides a quantitative assessment of the diversification component. Section 3 presents a development accounting exercise using a cross-section of countries in 1985. Concluding remarks are in Section 4.

2 Aggregate production function

In this section we derive the aggregate production function for a two-sector, three-factor economy. Our results can be easily extended to the more general case in which the number of sectors is smaller than the number of factors.[1] Consider a world composed of a large number of small open economies.

[1] When the number of sectors is larger than the number of factors, some of the sectors will not operate.

There are two goods, agriculture a and nonagriculture n, and three factors of production, physical capital K, effective labor H, and arable land T. Factors are mobile across sectors within a country, but internationally immobile. Goods can be traded internationally, and output prices p_a and p_n are determined in international markets.

A country can produce good i using a constant returns to scale production function $\overline{A}_i F^i(K_i, T_i, H_i)$, where $\overline{A}_i$ is the country-specific TFP of sector i. Function F^i is identical across countries. Assume that $F^i(K_i, T_i, H_i) = K_i^{\alpha_i^K} T_i^{\alpha_i^T} H_i^{\alpha_i^H}$ with $\alpha_i^K + \alpha_i^T + \alpha_i^H = 1$ and $0 < \alpha_i^f < 1$ for $f = \{K, T, H\}$. We assume agriculture to be land intensive in the sense that $\alpha_a^T \geq \alpha_n^T$.

2.1 Efficient level of production

Denote by $A_i \equiv p_i \overline{A}_i$ the price-adjusted TFP level in sector i. The central planner in each country chooses the allocation of factors across sectors that maximizes the aggregate level of output Y,

$$Y = G(K, T, H; \overrightarrow{A}) \equiv \max_{\{K_i, H_i, T_i\}_{i=a,n}} \left[A_a F^a(K_a, T_a, H_a) + A_n F^n(K_n, T_n, H_n) \right], \tag{1}$$

subject to,

$$K_a + K_n = K; \qquad H_a + H_n = H; \qquad T_a + T_n = T; \tag{2}$$

and given $\overrightarrow{A} \equiv [A_a, A_n]$. We assume that $\overline{A}_a$ and $\overline{A}_n$ are such that an efficient allocation is interior.

If functions F^i are identical across sectors ($F^i = F$), an interior solution requires the restriction $A_a = A_n$. As a result, the aggregate production function G takes the form $G(K, T, H; \overrightarrow{A}) = AF(K, T, H)$. To analyze the more general case in which $F^a \neq F^n$, denote $k \equiv K/H$ and $t \equiv T/H$, and define $B \equiv A_n/A_a$. Let $k_i^* \equiv K_i^*/H_i^*$ and $t_i^* \equiv T_i^*/H_i^*$ be optimal allocations for $i = \{a, n\}$. A property of these allocation rules is that they depend only on the ratios $(k,\ t;\ B)$ but not on the exact levels $(K, T, H; \overrightarrow{A})$, i.e., $k_i^* = k_i(k, t; B)$ and $t_i^* = t_i(k, t; B)$.

Proposition 1 For any $i = \{a, n\}$, efficient production satisfies

$$Y = G(K, T, H; \overrightarrow{A}) = \underbrace{A_i}_{\text{TFP}} \underbrace{F^i(K, T, H)}_{\text{Factors}} \underbrace{Z_i(k, t; B)}_{\text{Adjustment}} \tag{3}$$

where

$$Z_i(k, t; B) = \left(\frac{k_i^*}{k} \right)^{\alpha_i^K} \left(\frac{t_i^*}{t} \right)^{\alpha_i^T} \left[\alpha_i^K \frac{k}{k_i^*} + \alpha_i^T \frac{t}{t_i^*} + \alpha_i^H \right]. \tag{4}$$

Proof: Optimal allocations k_i^*, t_i^* for $i = \{a, n\}$ and h_a^* can be found using three first-order conditions of the form: $F_j^a(k_a^*, t_a^*, 1) = BF_j^n(k_n^*, t_n^*, 1)$ for $j = \{K, T, H\}$, and resource constraints: $k_a^* h_a^* + k_n^*(1 - h_a^*) = k$, and $t_a^* h_a^* + t_n^*(1 - h_a^*) = t$. In the Cobb-Douglas case, this five-equation system can be reduced to one nonlinear equation in h_a^*. Given h_a^*, the system of equations can be used to obtain k_i^*, t_i^*. Finally, for any sector i the aggregate production function can be written as

$$Y = G(K, T, H; \overrightarrow{A}) = A_i F^i(K, T, H)\left[\alpha_i^K \frac{F_K^i(K_i, T_i, H_i)}{F_K^i(K, T, H)} + \alpha_i^T \frac{F_T^i(K_i, T_i, H_i)}{F_T^i(K, T, H)} + \alpha_i^H \frac{F_H^i(K_i, T_i, H_i)}{F_H^i(K, T, H)}\right],$$

which in the Cobb-Douglas case has the form specified in (3) and (4).

The main virtue of equation (3) is that it provides a multiplicative formulation of aggregate production in three fundamental components. The first two are the traditional TFP and factors components. They describe the total production of the economy if all of factors are placed in only sector i, a "reference" sector. Such decision is suboptimal since, by assumption, the solution is interior. Term Z_i, which is necessarily larger than 1, adjusts the output implied by the first two terms to obtain the efficient level of output. Thus, Z_i captures gains from diversification.

2.2 Solow residuals

The typical decomposition of the sources of cross-country income differences (Klenow and Rodriguez-Clare 1997; Hall and Jones 1999) assumes an aggregate production of the form

$$Y = AK^\alpha T^\beta H^{1-\alpha-\beta}, \tag{5}$$

where $\alpha \simeq 1/3$, β is typically set to zero, and A is the Solow residual. A limitation of using (5) for cross-country comparisons is that it may not describe well the production possibilities of agricultural countries, in which land plays a more central role than capital. The aggregate production function of these countries is better described by (3).

In order to make (3) consistent with extensive growth accounting literature that regards (5) as a satisfactory description of industrialized countries, we make assumptions to guarantee that (3) is approximately equal to (5) for countries in which agriculture is a smaller share of output. This is guaranteed by choosing nonagriculture as the "reference" sector, and by assuming that the nonagricultural production function is given by (5). Under these conditions, (3) can be written as

$$Y = A_n \cdot Z_n \cdot K^{\alpha} T^{\beta} H^{1-\alpha-\beta}, \tag{6}$$

which combined with (5) implies

$$A = A_n \cdot Z_n \left(k, t; B\right). \tag{7}$$

This equation provides two main insights. First, Solow residuals are a re-scaled version of nonagricultural TFPs. Second, the size of the re-scaling, Z_n, is not identical for all countries and it depends on factor endowments. If, as we confirm below, Z_n is larger for poorer than richer countries, then Solow residuals underestimate underlying productivity differences between rich and poor countries in the nonagricultural sector.

2.3 Quantitative exercise

In this section we compute Z_n for a sample of countries. We construct $p_a Y_a$ and $p_n Y_n$ following Caselli (2005), who uses FAO 1985 data to measure PPP agricultural output.[2] Aggregate K is computed using the perpetual inventory method on investment data from Penn Word Tables. Average human capital $h = H/L$ is measured using schooling years from Barro and Lee (2000), and following Hall and Jones (1999) to adjust for Mincer returns. We use data on arable land T and labor force in agriculture L_a from the World Bank.

Given a specific set of inputs shares α_i^f, we use the efficiency conditions and resource constraints of the model to compute K_i, T_i, and H_i. In addition, Z_n is obtained using equation (4). Regarding input shares, we consider two scenarios. First, we assume as most accounting exercises that labor share is 2/3 for all countries ($\alpha_a^H = \alpha_n^H = 0.66$). We also assume that agriculture is "land-labor intensive" and nonagriculture is "capital-labor intensive" and set $\alpha_a^K = \alpha_n^T \simeq 0$. Although these assumptions better describe agricultural technologies in poorer economies, agriculture is a very small fraction of output in richer countries.

Figure 1 portrays the corresponding Z_n relative to the US in 1985 at different levels of income. Two observations emerge. First, Z_n is larger in poor than in rich countries. For example, Z_n is around 1.15 for Uganda. This means that the technological gap between Uganda and US in

[2] See p. 720-721.

nonagriculture is 15% larger than what is implied by Solow residuals. For the other very poor countries, this number ranges between 3 and 7%. This implies that the dispersion of nonagricultural TFPs is larger than the dispersion of Solow residuals. The second conclusion from Figure 1 is that Z_n quickly decreases with the level of income, and for most countries outside the poorest range, it is in the order of 1%.

Consider now a second scenario that relaxes the assumption of an equal labor share in both sectors. Gollin (2002) reports that in most countries, regardless of their level of development, $\alpha_a^H < \alpha_n^H$. For instance, for the US in 1992, $\alpha_a^H \simeq 0.25$ while $\alpha_n^H \simeq 0.7$. We find that if $\alpha_a^H = 0.33$ and $\alpha_a^T = 0.69$, then Z_n is significantly larger than in the first scenario. Specifically, it is around 1.34 in Uganda, and between 1.08 and 1.17 for most other poorer countries. In sum, our exercises suggest that very poor countries are not properly represented in standard models that abstract from land and agriculture.

3 Development accounting

Traditional development accounting exercises write (5) as

$$y \equiv \frac{Y}{L} = \widetilde{A} \cdot \widetilde{X} \tag{8}$$

where $\widetilde{X} \equiv (K/Y)^{\frac{\alpha}{1-\alpha-\beta}} (T/Y)^{\frac{\beta}{1-\alpha-\beta}} h$, and $\widetilde{A} \equiv A^{\frac{1}{1-\alpha-\beta}}$. The analogous formulation for (6) is

$$y \equiv \frac{Y}{L} = \overline{A} \cdot \overline{X} \cdot \overline{Z}. \tag{9}$$

where $\overline{X} \equiv (K/Y)^{\alpha_n^K/\alpha_n^H} (T/Y)^{\alpha_n^T/\alpha_n^H} h$ is the factor intensity, $\overline{A} \equiv A_n^{1/\alpha_n^H}$ is TFP, and $\overline{Z} \equiv Z_n^{1/\alpha_n^H}$ is the diversification component. Following Klenow and Rodríguez-Clare (1997), the contributions of factors, TFP, and the diversification components to output dispersion can be defined as:

$$\Phi_{\overline{X}} = \frac{cov\left(\ln y, \ln \overline{X}\right)}{var\left(\ln y\right)}; \quad \Phi_{\overline{A}} = \frac{cov\left(\ln y, \ln \overline{A}\right)}{var\left(\ln y\right)}; \quad \Phi_{\overline{Z}} = \frac{cov\left(\ln y, \ln \overline{Z}\right)}{var\left(\ln y\right)}.$$

where $\Phi_{\overline{X}} + \Phi_{\overline{A}} + \Phi_{\overline{Z}} = 1$.

Since $\overline{Z}$ is larger for poorer countries (Figure 1), then $\Phi_{\overline{Z}}$ is negative. According to equation (7), standard accounting measures the contribution of TFP as $\Phi_{\overline{A}} + \Phi_{\overline{Z}}$, and as a result, it downplays the role of TFP. Equation (4) suggests that since Z_n depends on factor endowments, then part of $\Phi_{\overline{Z}}$ should be added to $\Phi_{\overline{X}}$. Since it is unclear how to exactly assign $\Phi_{\overline{Z}}$, we divide $\Phi_{\overline{Z}}$ equally between TFP and factors components.[3] Specifically, we define the contributions of factors, Φ_X, and TFP, Φ_A, as

$$\Phi_X = \Phi_{\overline{X}} + \Phi_{\overline{Z}}/2; \quad \Phi_A = \Phi_{\overline{A}} + \Phi_{\overline{Z}}/2.$$

Table 1 reports development accounting for the two-sector economy. For nonagriculture we use the standard values $\alpha_n^H = 0.66$ and $\alpha_n^K = 0.33$. The first row reproduces results for a one-sector model, according to which TFP accounts for 64% of the variance of output per worker. This percentage increases slightly to 67% in our two-sector model. The relative small increase in the contribution of TFP is due to the fact that, except for the very poor countries, the adjustment factor Z_n is quantitatively small.

4 Concluding remarks

This paper derives a representation of the aggregate production function in a two-sector model. The representation has two special attributes: first, it is suitable for variance decomposition; and second, it explicitly derives the diversification component Z_n for the case of Cobb-Douglas technologies. Although theoretically, the presence of this diversification effect should increase the role of TFP in explaining cross-country income differences, we find the quantitative effects to be small. However, the diversification component is large for the group of very poor countries. Research in understanding the special characteristics of these very poor economies is warranted.

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Figure 1: Diversification term Z - 1985

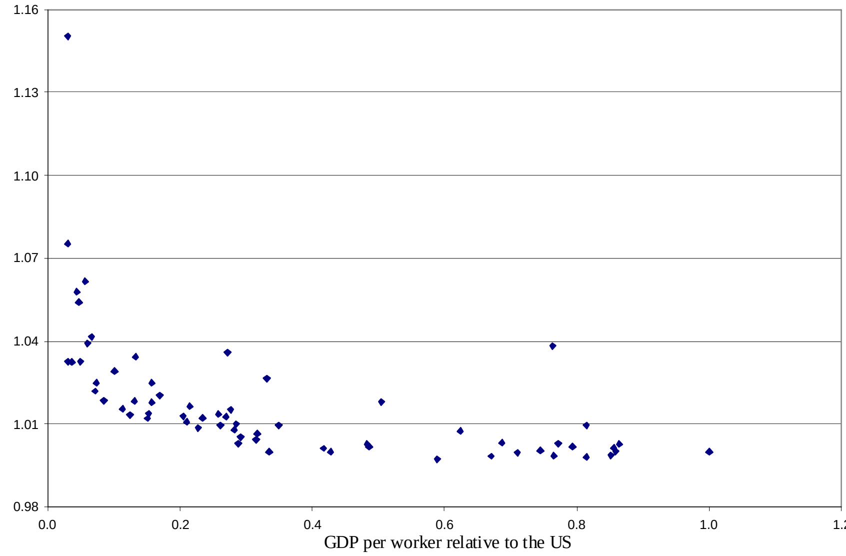

Table 1. Development accounting

α_a^K	α_a^T	α_a^H	$Factors$	TFP
0.33	0.01	0.66	36%	64%
0.01	0.33	0.66	35%	65%
0.01	0.69	0.30	33%	67%