

Introduction to Dynamic Stochastic General Equilibrium Models¹

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Preface

This book is an introduction to dynamic stochastic general equilibrium models. It does not require any previous knowledge of macroeconomics, but it requires a good intermediate microeconomics course, a good calculus course, and some knowledge of Microsoft Excel and Matlab.

The book has in mind a new generation of economists who regard macroeconomics as applied Dynamic Stochastic General Equilibrium theory (DSGE). Two distinguishing features of this approach to macroeconomics are: (i) its insistence providing well micro-founded internally-consistent stories; and (ii) its focus on deriving overall qualitative, but perhaps more importantly, quantitative implications of the theories, and testing those predictions against actual data.

The book differs significantly from all existing macroeconomic textbooks. Its focus is on the economics rather than on the math or the econometrics. Chapters are written in a way that is familiar to DSGE practitioners. Each chapter poses a fully dynamic general equilibrium model, defines and characterizes the equilibrium and its qualitative properties. Then it investigates the ability of the model to explain actual data by collecting and discussing relevant data, and deriving stylized facts. The model is calibrated, tested, and then utilized to perform experiments and policy analysis. Finally, the chapter provides extensions and robustness checks.

The repetitive application of this methodology trains students in all aspects of modern macroeconomics and helps develop critical thinking in an organized and constructive way. As a result, the book goes beyond the standard goal of teaching theories and concepts, to actually give students the tools required for their own advancement. In particular, by the end of this course students are better prepared to read and understand macroeconomic articles appearing in leading economic journals.

The computational exercises are an integral part of the book, as is the case in modern macroeconomics. Quantitative versions of the theories are implemented in Excel and Matlab, well-known softwares that are widely available and sufficiently powerful for simple but also complex applications. For example, chapters 8 and 12 show how to solve Ramsey problems and optimal taxation problems in Excel.

This book also avoids the standard practice of separating the topics of economic growth and business cycles into different chapters. Instead, the book focuses on the simultaneous performance of the theory both along the

business cycle frequency but also the medium and long term frequencies. This integrated approach makes a lot of sense. For example, Chapter 2 calibrates a simple Robinson Crusoe economy in two different ways, one matching trend components and another matching business cycle components. Students learn about the key issue of identification: models can tell completely different but plausible stories about the underlying determinants of business cycles depending on how parameters are identified. They discover first hand the origins of many economic controversies. This approach also implies that I avoid using the HP filter, a popular tool among practitioners, because it separates the trend from the cycle. However, I also present and discuss results when the HP filter is utilized.