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U.S Inequality: Debt Constraints or Incomplete Asset Markets?

Juan-Carlos Cordoba[a*†]

[a] Rice University

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Abstract

To examine the role of debt constraints and incomplete asset markets (lack of insurance markets) in explaining U.S. inequality, we run horse races between competing models. For a widely used model, we decompose inequality into its fundamental driving forces. The underlying source of inequality in all models is uninsurable idiosyncratic risk. Both debt constraints and incomplete asset markets are needed to account for inequality, but asset market incompleteness is the key friction. It better accounts for the concentration and dispersion of wealth, and is the most costly friction in terms of welfare. Tight debt constraints are important for explaining the lower tail of the wealth distribution.

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*Corresponding author: jcordoba@rice.edu

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1. Introduction

Two distinctive features of the wealth distribution in the U.S. are its large concentration and dispersion. For example, according to the 1992 Survey of Consumer Finance, the wealthiest 1% of the population owns over 30% of the nation's wealth, while the poorest 50% owns less than 5%. Moreover, Rodríguez *et al.* (2002, Table 1) reports that the standard deviation of wealth is more than six times its mean. This large dispersion suggests that the welfare gains of even a small reduction in inequality may be larger than those of eliminating business cycles or increasing the growth rate of the economy (Córdoba and Verdier 2007). What are the *main* causes of this large wealth inequality?

A growing literature regards long term inequality as primarily the result of two market frictions: debt constraints and incomplete asset markets[1]. Intuitively, debt constraints are limits to the amount that individuals can borrow using a particular asset (e.g., credit card limits) or a set of assets (e.g., credit cards plus mortgages, etc.). Asset market incompleteness refers to restrictions on the type of assets available in the market (e.g., only a riskless bond). Households facing these frictions cannot fully diversify idiosyncratic risk, and as a result, are driven into poverty and richness randomly.

This paper provides a quantitative assessment of the separate role of these two frictions in explaining U.S. inequality. This evaluation is important for at least two reasons. First, each market friction likely results from different underlying constraints. Debt limits typically arise from commitment and enforcement problems while asset market incompleteness likely arises from asymmetric information and moral hazard problems, although these frictions are not necessarily mutually exclusive. By shedding light on the underlying main determinants of inequality, this paper provides direction for researchers and policymakers. From a policy perspective, the paper sheds light on the benefits, in terms of equity, of policies oriented

[1] Papers include Aiyagari (1994), Huggett (1996), Quadrini (1997), Castañeda, et al. (2003), De Nardi (2004), Heatcote et al. (2004), and Domeij and Heathcote (2004).

toward improving credit access, such as government promoted loans for education and home purchase, or more restricted bankruptcy laws, relative to policies oriented toward improving insurance access, such as unemployment, health and disability insurance, or progressive taxation.

The second reason the evaluation is important is that recent literature has argued that asset market incompleteness may play a secondary role in explaining key regularities in the data, including features of the wealth and consumption distributions. This view is supported by the quantitative results of Heaton & Lucas (1996), who find small effects of market incompleteness on asset prices, and by the theoretical results of Levine & Zame (2002) and Kehoe & Levine (2001), who show that asset market incompleteness does not matter if agents are sufficiently patient, and that models with only debt constraints are simpler and produce results similar to standard incomplete market models. On the issue of inequality, Krueger & Perri (KP, 2002) argue that debt-constrained models[2] can better explain the rising inequality in the U.S. In contrast to this literature, this paper shows that asset market incompleteness matters more than debt constraints in explaining inequality.

The core of the paper is a flexible model that can incorporate different types and degrees of market imperfections. A parameter $\Gamma \in [0, 1]$ controls the extent of asset market incompleteness, and a parameters $\Psi \in [0, 1]$ controls the extent of credit access. If $\Gamma = 1$, a full set of Arrow securities is available in the market, but only a riskless bond is available if $\Gamma = 0$. Similarly, if $\Psi = 1$, net debts are restricted only by a "natural" limit, defined as the maximum amount that an individual can possibly repay, but if $\Psi = 0$ net debts are precluded, or equivalently, debts must be secured by collateral.

This flexible model accommodates different asset structures: $(\Gamma, \Psi) = (1, 1)$ is a frictionless or complete markets economy; $(\Gamma, \Psi) = (0, \Psi)$ with $1 \geq \Psi \geq 0$ are standard incomplete

[2]Following Kehoe & Levine (2002), we call these models debt-constrained models.

market (SIM) economies[3]; and $(\Gamma, \Psi) = (1, \Psi)$ with $1 > \Psi \geq 0$ are debt-constrained (DC) economies with secured and unsecured debt[4]. All models with either $\Gamma < 1$ or $\Psi < 1$ are formally models of *incomplete markets*. Hence, the analysis essentially focuses on the effects of different sources of incompleteness.

More specifically, the paper assesses the quantitative performance of different market structures in terms of their ability to replicate key features of the U.S. inequality, such as wealth concentration and wealth dispersion. To assess the robustness of the findings, the equilibrium of each economy is computed for three different set of parameters used in three influential papers: Aiyagari (1994), KP (2002), and Castañeda, Días-Giménez & Ríos-Rull (CDR, 2003) respectively.

The main finding is that SIM models substantially outperform DC models in accounting for wealth inequality. Although all models fail to reproduce the wealth concentration or dispersion found in the data, DC models perform particularly poorly. An extreme case occurs using Aiyagari's calibration. While SIM models can produce significant inequality, perfect equality is an equilibrium in the DC models under this calibration. The reason is that the modified golden rule level of capital stock, equally distributed across agents, is enough to secure all trading in Arrow securities required for perfect risk sharing and perfect equality in DC economies. An interpretation of this result is that *all* inequality in Aiyagari's economy is rooted in the asset market incompleteness, *none* in the debt limit.

For the other two calibrations, perfect equality is not an equilibrium in DC models. In fact, the Gini coefficients of wealth and consumption are similar to those of SIM models. However, DC models can account for less than 25% of the variation of wealth while SIM models can account for more than 60%. Moreover, DC models consistently fail to explain the large concentration of wealth in the top tail of the distribution. While the wealthiest

[3] Aiyagari (1994) considered the special cases $\Psi = 0$ and $\Psi = 1$.

[4] The case $\Psi = 0$ corresponds to secured debt, in the spirit of Kiyotaki and Moore (1997). The case $\Psi > 0$ also allows for unsecured debt, in the spirit of KP (2002). See details in section 3.5.1.

1% of the U.S. population owns close to 30% of the nation's wealth, DC models predict less than 7% concentration.

These results have a clear intuition. Debt-constraints mainly affect agents in the lower and middle part of the wealth distribution, agents who are either constrained or are more likely to be constrained in the near future. Agents in the upper part of the distribution are unlikely to be constrained in the future and only a small fraction of their wealth can be explained by this possibility. Moreover, wealthy agents in DC models actually tend to dissipate wealth because the equilibrium interest rate is below the rate of time preference and market insurance is available, which makes it harder for these models to explain the existence of very wealthy agents. By contrast, if insurance markets are missing then even rich agents may further accumulate wealth to isolate consumption fluctuations as much as possible from income fluctuations.

2. Related Literature

In addition to the literature mentioned above, our paper is particularly related to KP (2002). They study a DC economy where debt is backed by the threat of exclusion from spot markets. They find that their model better explains recent trends in U.S. inequality regarding a substantial rise of income inequality but only a moderate rise of consumption inequality. They argue that this evidence can be explained by an exogenous increase of income risk that makes default more costly, relaxes debt constraints, and improves risk sharing.

Our assessment of DC models is the opposite. We find that DC models fail to capture the two main features of inequality: wealth dispersion and wealth concentration in the top tail. KP are silent about these two key predictions of their model. They focus on consumption inequality, which is substantially less dispersed and concentrated than wealth. We show that DC and SIM models can produce similar predictions on consumption inequality, but SIM models substantially outperform DC models in accounting for wealth inequality.

Furthermore, KP focus on the role of unsecured debt. However, as we show in the paper, models with unsecured debt counterfactually predict large negative wealth levels for a significant fraction of the population. However, in the data the fraction of population with negative wealth is close to zero. Moreover, most debts in the U.S. are secured. For example, Canner *et al.* (1995, Figure 1) report that more than 75% of household debt in the U.S. is mortgage debt. A large fraction of the remaining part is also secured because it includes loans for automobiles, mobile homes, trailers, etc. Even a large share of seemingly unsecured debt, such as credit card debts, may implicitly be secured since debtors typically own assets such as cars or houses that might be seized to discharge the debt.

The remaining part of the paper is divided into three sections. Section 3 describes the models and their equilibrium properties, focusing on key distinguishing properties of debt-constrained models; Section 4 calibrates the models and report the quantitative findings; Section 5 concludes.

3. Model economy

This Section sets up a parsimonious model that can be used to study different types of credit-constrained and incomplete asset markets economies. The economies are populated by a continuum of infinitely lived individuals of mass one and time is discrete.

3.1. Employment opportunities

Individuals face a random endowment, e_t, of efficiency labor units. Labor endowments are independently and identically distributed across households and known by agents at the beginning of the period. They follow a finite state Markov chain with conditional transition probabilities given by $\pi(e'|e) = \Pr(e_{t+1} = e'|e_t = e)$, where e and $e' \in E \equiv \{e_1, e_2, ..., e_n\}$,

$0 < e_1 < ... < e_n$, and $\pi(e_1|e) > 0$ for all $e \in E$. π has a unique invariant distribution, and the unconditional expected value of endowments is 1. Notice that $e_1 < 1$ and $e_n > 1$ is implied by this assumption.

3.2. Preferences

Preferences over consumption streams are described by:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right], \ 0 < \beta < 1,$$

where E_t is the mathematical expectation conditional on information available to the agent up to time t, c_t is consumption, and u satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

3.3. Production possibilities

There is a freely available constant returns to scale production technology that transforms efficiency units of labor, L, and capital, K, into output according to the function $F(L, K)$. Aggregate capital is obtained by aggregating the capital holdings, and aggregate labor by aggregating the efficiency labor units, supplied by every household. Capital depreciates geometrically at a constant rate δ. Define gross output as $f(k) \equiv F(k, 1) + (1 - \delta)k$ and assume that f satisfies $f(0) = 0$, $f' > 0$, $f'' < 0$.

3.4. Market arrangements

Two sequential market arrangements will be considered: one without insurance markets and one with insurance markets. In both arrangements, individuals can accumulate wealth in the form of a riskless asset, b_t. This is the only asset available in the first arrangement. In the second arrangement, agents can also trade one period-Arrow securities which are claims to consumption goods contingent on the realization of the individual's endowments, $a_t(e_{t+1})$.

Agents are subject to debt limits to be described in detail below. Finally, markets are competitive. Firms rent factors of production from households in competitive spot markets, and assets are traded in competitive spot markets. These assumptions imply that factor prices are given by the corresponding marginal productivities and that insurance prices are actuarially fair.

3.5. Recursive competitive equilibrium

For simplicity, consider only stationary situations in which aggregate variables are constant. Denote r the rental price of capital, w the wage rate, and $q(e_j, e_i)$ the price of an Arrow security[5] that pays one unit of good in state e_j at time $t+1$ if the state at time t is e_i. Let x_t denote an individual's total resources available at time t defined as $x_t \equiv we_t + (1+r)b_{t-1} + a_{t-1}(e_t)$, where we_t is wage income, $(1+r)b_{t-1}$ is either riskless assets or riskless debt inclusive of interest, and $a_{t-1}(e_t)$ is the realized value of insurance claims or liabilities. The following is a recursive formulation of the problem.

3.5.1. Individual problem

An individual's state at time t is described by the couple $(x, e) \in X \times E$. The following is the dynamic program solved by an individual:

$$V(x,e) = \max_{c \geq 0, b, \{a(e_i)\}_{i=1}^n} \left\{ u(c) + \beta \sum_{i=1}^n V\left(we_i + \Gamma a(e_i) + (1+r)b, e_i\right) \pi(e_i|e) \right\}, \tag{1}$$

subject to:

$$\begin{aligned} x &\geq c + b + \Gamma \sum_{i=1}^n a(e_i) q(e_i, e), \\ \Psi \times \phi^\Gamma(e_i) &\geq -\left(\Gamma a(e_i) + (1+r)b\right) \text{ for } e_i \in E. \end{aligned}$$

[5]The use of the term "Arrow security" is justified by Proposition 3 below, which shows that these assets can support perfect risk sharing.

The first restriction of the problem states that resources can be used for consumption, accumulation (or borrowing) of riskless bonds, b, and purchases (or sales) of contingent claims, $a(e_i)$. The parameter $\Gamma \in [0,1]$ determines the extent of asset-markets incompleteness. We only consider the two extreme cases: $\Gamma = 1$ (full insurance markets) and $\Gamma = 0$ (no insurance markets).

The second constraint of the problem is the debt limit. An individual's total financial liabilities at any period, $-\Gamma a(e_i) - (1+r)b$, cannot exceed the limit $\Psi \times \phi_i^\Gamma$ where $\Psi \in [0,1]$ is a parameter determining the extent of the credit friction. The term $\phi^\Gamma(e_i)$ is the "natural" debt limit. For the case $\Gamma = 0$, $\phi^0(e_i)$ is the maximum amount of debt that an individual can possibly repay (a.s). Assuming $r > 0$, $\phi^0(e_i) \equiv \frac{we_1}{r}$ (see Aiyagari 1994 for details). For the case $\Gamma = 1$, the natural debt limit is the present value of the labor income endowment:

$$\phi^1(e_i) = w \sum_{s=0}^{\infty} \sum_{e^s|e_0=e_i} p^s(e^s) e_s \tag{2}$$

where $e^s = [e_0, e_1, ..., e_s]$ and $p^s(e^s) = q(e_s, e_{s-1})...q(e_2, e_1)q(e_1, e_0)$ for $s > 0$ and $p^s(e^s) = 1$ for $s = 0$ (see Ljungqvist and Sargent, 2004, pg 225 for details). We mainly consider the extreme cases $\Psi = 1$, maximum borrowing, and $\Psi = 0$ no borrowing.

We call economies with $\Gamma = 0$ standard incomplete markets (SIM) economies, and economies with $\Gamma = 1$ debt-constrained (DC) economies. Notice that for the case $(\Gamma, \Psi) = (1, 0)$ the debt limit becomes $a(e_i) \geq -(1+r)b$ so that households can assume contingent debts up to the value of their uncontingent assets. Thus, short-sales of Arrow securities must be collateralized or contingent debts must be *secured*. Unsecured debt arises when $\Psi > 0$.

The solution to the household problem is a set of functions that map states into choices for consumption and asset demands: $\{c(x,e), b(x,e), a(e_1; x, e), ..., a(e_n; x, e)\}$.

3.5.2. Equilibrium concept

Each period the economy-wide state is a measure of households, J_t, defined over $\mathcal{B}$, an appropriate family of subsets of $\{X \times E\}$. This paper focuses on a stationary situation in which J_t is constant over time and the aggregate labor supply is equal to 1.

Definition: A stationary equilibrium is a value function, $V(x, e)$; policy functions $\{c(x, e), b(x, e), a(e_1; x, e), ..., a(e_n; x, e)\}$; a probability measure of households, J; factor prices (r, w); asset prices $q(e_j, e_i)$ for $i, j = 1, 2, .., n$; and aggregate capital K such that:

1. $c(x, e)$, $b(x, e)$, and $\{a(e_i; x, e)\}_{i=1}^{n}$ are optimal decision rules given prices, and $V(x, e)$ solves the functional equation (1.)

2. Factor prices equal their marginal productivities: $r = F_1(K, 1) - \delta$ and $w = F_2(K, 1)$.

3. Insurance prices are actuarially fair: $q(e_i, e_j) = \pi(e_i, e_j)/\,(1 + r)$ for e_i,$e_j \in E$.

4. $K = \int_{X\times E} b(x, e)dJ(x, e)$.

5. The measure of households is stationary:

$$J(B) = \int_B \left\{ \int_{X,E} I_{x'=we'+\Gamma a(e';x,e)+(1+r)b(x,e)}\pi(e', e)dJ(x, e) \right\} dx'de'$$

   for all $B \in \mathcal{B}$; I is an indicator function.

3.6. Characterization of the equilibrium

This section characterizes the equilibrium. The case $(\Gamma, \Psi) = (1, 1)$ describes a complete markets economy. In this case, the capital stock, K^*, is determined by the modified golden rule and the interest rate equals the rate of time preference, $\rho \equiv 1/\beta - 1$:

$$1 = \beta f'(K^*, 1) \text{ and } r^* = \rho.$$

The cases $(\Gamma, \Psi) = (0,0)$ and $(\Gamma, \Psi) = (0,1)$ describe two SIM economies. They are analyzed by Aiyagari (1994) and Hugget (1997) among others. The steady state capital stock in these economies is larger than K^* and the interest rate is below the rate of time preference.

The rest of this sub-section characterizes the case $(\Gamma, \Psi) = (1,0)$. To our knowledge, most of the results that follow are new to the literature. It is convenient to describe first the solution to the individual problem given that the interest rate is below the rate of time preference ($r < \rho$) and insurance prices are actuarially fair. This partial equilibrium problem is an extension of *the income fluctuation problem* (Schechtman and Escudero 1977). Second, we study the determination of the interest rate. We provide a necessary and sufficient condition that ensures in fact $r < \rho$ in equilibrium.

3.6.1. The income fluctuation problem with insurance markets

For analytical convenience we only consider the case of *i.i.d* endowments in this subsection. We drop this assumption for the general equilibrium characterization and the quantitative work. As a result of this assumption, the only relevant individual state variable is x since e has no informational content.

Let $(\Gamma, \Psi) = (1,0)$, $\underline{x}_i \equiv we_i$ and suppose $-1 < r < \rho$ and $q(e_i) = \pi(e_i)/(1+r)$. The individual problem can be written as:

$$V(x) = \max_{s \geq 0} \{u(x-s) + \beta W(s)\} \tag{SP1}$$

where

$$\begin{aligned} W(s) &= \max_{x'_i \geq \underline{x}_i} \sum_{i=1}^{n} V(x'_i)\pi(e_i) & \text{(SP2)} \\ \text{subject to } s(1+r) &\geq \sum_{i=1}^{n} (x'_i - \underline{x}_i)\, \pi(e_i) \end{aligned}$$

This formulation breaks the problem down into two simpler sub-problems: a deterministic-dynamic problem and a stochastic-static problem. The first sub-problem describes the optimal saving-consumption decision, which only requires a solution for the *total* amount of savings, s. The second sub-problem describes the optimal allocation of savings across different state claims.

Consider first the solution to the second sub-problem. Standard arguments can be used to show that the first order condition and envelope conditions of this problem are:

$$\frac{\partial V(x_i'^*)}{\partial x} \leq \lambda, \text{ with equality if } x_i'^* > \underline{x}_i \text{ and } \frac{\partial V(x_i'^*)}{\partial x} = \frac{\partial u(c')}{\partial c},$$

where λ is the multiplier on the resource constraint. Notice that if the debt limit is not binding then it is optimal to equate resources across states at the level $s(1+r)+\omega$, where $\omega \equiv \sum_{i=1}^{n} \underline{x}_i \pi(e_i)$.

Figure 1 illustrates the solution of the second sub-problem for $n = 4$. The solution is simple. Savings s have to be allocated to smooth resources across states as much as possible. This is accomplished by allocating resources to the poorest state until there are two poorest states, then allocate resources into the two poorest states equally until there are three poorest states, and so on. Specifically, if $s = 0$ no insurance is purchased at all so that $x_i'^* = \underline{x}_i$ for all i. For savings between 0 and s_2, all savings must be allocated to the poorest state. Thus, it is optimal to choose $x_1'^* = \frac{s(1+r)+\underline{x}_1\pi_1}{\pi_1}$ and $x_j'^* \equiv \underline{x}_j$ for $1 < j \leq n$. The threshold level of savings s_2 is obtained when $x_1'^* = \underline{x}_2$. For savings between s_2 and s_3, it is optimal to choose $x_1'^* = x_2'^* \equiv \frac{s(1+r)+\pi_1\underline{x}_1+\pi_2\underline{x}_2}{\sum_{h=1}^{2}\pi_h}$ and $x_j'^* \equiv \underline{x}_j$ for $2 < j \leq n$. One can proceed in this fashion to find the solution for successively higher savings. The following proposition summarizes the solution described in Figure 1.

Proposition 1 *Define* $s_i \equiv \frac{1}{1+r}\sum_{j=1}^{i}\left(\underline{x}_i - \underline{x}_j\right)\pi_j$ *for* $i = 1,..,n$. *Then, (i) for* $s_i \leq s \leq s_{i+1}$,

$$\begin{aligned} x_j'^{*} &= x_j(s) \equiv \frac{s(1+r)+\sum_{h=1}^{i}\pi_h \underline{x}_h}{\sum_{h=1}^{i}\pi_h} \text{ for all } j \leq i, \\ x_j'^{*} &= x_j(s) \equiv \underline{x}_j \text{ for } i < j \leq n, \end{aligned}$$

and (ii) for $s \geq s_n$, $x_j'^{*} = s(1+r)+\omega$ *for all* j.

This closed form solution is very convenient for analytical and computational purposes. At least for $i.i.d$ endowments, the savings problem with many assets is just as complicated as the one with a single asset[6].

Consider now the solution to first sub-problem. Denote $s = g(x)$ the solution of this problem. It is standard to show that g is an increasing function of x. The law of motion of x is obtained by combining $g(x)$ and Proposition 1. Figure 2 provides a graphical description of the solution (all proofs are in the Appendix). Define $\widetilde{x} \equiv g^{-1}(s_n)$. Households with resources $x \geq \widetilde{x}$ save enough to fully smooth resources for the next period so that x' is a deterministic function of x, and $x' < x$ because $r < \rho$. Households with resources $x < \widetilde{x}$ do not fully insure. Instead, with some positive probability $x' = \underline{x}_n$. Moreover, if x falls below a certain level[7] $\widehat{x}$ $(> \underline{x}_1)$, agents purchase no insurance at all and $x' = \underline{x}_i$ with probability π_i. The ergodic set for x is $[\underline{x}_1, \underline{x}_n]$.

3.6.2. General equilibrium

This section shows that $r < \rho$ in a stationary equilibrium of the economy $(\Gamma, \Psi) = (1, 0)$ provided that a necessary and sufficient condition is satisfied. The condition guarantees that the debt limit is binding for a positive mass of agents. Endowments are not required to be

[6]The computational algorithm used to solve the general case is motivated by Figure 1 and Proposition 1. One needs to find $n \times n$ threshold levels of resources that solve the equations $V'(\underline{x}_i, e_i) = V'(\underline{x}_j, e_j)$, and then compute the optimal levels of resources, $x_j'^{*}$, as linear functions of those thresholds and s. Details are available from the Appendix.

[7]$\widehat{x}$ is defined by the equation $u'(\widehat{x}) = \beta\frac{\partial W(0)}{\partial s}$.

i.i.d in this section.

Consider the amount of contingent assets that a representative household in a complete markets economy would need to buy or sell in order to sustain a constant consumption profile equal to the modified golden rule level of consumption. The proof of the next Proposition shows that $a(e) = -w^* \cdot \theta(e)$, where $\theta(e) = \sum_{s=0}^{\infty} \beta^s \left(E[e_s - 1|e]\right)$ and $w^* = F_2(K^*, 1)$. Notice that $w^* \cdot \theta(e)$ is the expected present value of labor income above the average labor income. Thus, the representative household would like to make $a(e)$ negative in the good states in exchange for making $a(e)$ positive for the bad states. The maximum level of contingent debt is thus given by $-w\overline{\theta}$ where $\overline{\theta} = \max_e \{\theta(e)\}$. Is this level of debt below the debt limit? The following Proposition (proved in the Appendix[8]) states that it is if the modified golden rule level of capital is large relative to a measure of labor income risk in the economy.

Proposition 2 *A stationary equilibrium with perfect risk sharing exists if and only if*

$$\frac{K^*}{f(K^*)} \geq \frac{\beta\overline{\theta}}{1+\overline{\theta}}. \tag{3}$$

To illustrate this Proposition consider two cases. First, suppose shocks are *i.i.d.* In that case, $\theta(e) = e - 1$ and condition (3) reads $\frac{K^*}{f(K^*)} \geq \beta\frac{e_n - 1}{e_n}$. In words, perfect risk sharing is possible if and only if the best possible endowment is not extremely large relative to the capital stock. Intuitively, agents would like to borrow against their best possible endowment. If such endowment is large relative to the average, then the capital in the economy may not be enough to secure all required borrowing. Second, suppose $E_t[e_t - 1|e] = \tau^t\,(e-1)$ so that $\tau \in [0, 1]$ captures the degree of persistence[9]. In that case $\theta(e) = \frac{e-1}{1-\beta\tau}$ and perfect risk sharing is an equilibrium if $\frac{K^*}{f(K^*)} \geq \beta\frac{e_n - 1}{e_n - \beta\tau}$. This expression shows that persistence may prevent perfect risk sharing.

[8] The Appendix is available at http://www.ruf.rice.edu/˜jcordoba/usi_apdx.pdf.

[9] This process does not fit our assumption of a discrete Markov process but it helps to develop some intuition.

Finally, the following Proposition (proved in the Appendix) shows that $r < \rho$ if (3) does not hold. Denote by K^b and r^b the equilibrium capital stock and interest rate of the DC economy $(\Gamma, \Psi) = (1, 0)$.

Proposition 3 *Suppose* $\frac{K^*}{f(K^*)} < \frac{\beta\overline{\theta}}{1+\overline{\theta}}$. *Then* $K^b > K^*$ *and* $r^b < \rho$.

4. Quantitative Evaluation

This section assesses the performance of five model economies under three plausible parametrizations. Two models are SIM models ($\Gamma = 0$) with either a zero ($\Psi = 0$) or a natural debt limit[10] ($\Psi \approx 1$), two DC models ($\Gamma = 1$) with either a zero or a natural debt limit. This last model is actually a complete markets economy. Finally, we consider a DC model with an intermediate debt limit. In particular, we choose the natural debt limit of the SIM model, ϕ^0.

The three parametrizations considered are versions of: (i) Aiyagari (1994); (ii) KP (2002); and (iii) CDR (2003). These parametrizations differ substantially from each other and serve to assess the robustness of the results. In all models, a period refers to one year, preferences are isoelastic, $u(c) = \frac{c^{1-\mu}}{1-\mu}$, and the production function is Cobb-Douglas, $F(K, L) = K^{\alpha}L^{1-\alpha}$. Details about the computational procedure are provided in the Appendix.

4.1. Calibration 1: Aiyagari (1994)

Aiyagari (1994) assumes $\alpha = 0.36$, $\delta = 0.08$ and $\beta = 0.96$. He considers three values for the parameter μ: 1, 3 and 5. We only use his intermediate value, $\mu = 3$. For the endowment process, Aiyagari follows Heaton and Lucas (1996) who estimate the autoregressive repre-

[10] For technical reasons, the cases $\Psi = 1$ are actually approximated by setting $\Psi = 0.96$. Ψ close to 1 results in paths where consumption is close to zero and utility goes to $-\infty$. In some cases this slows down the convergence of the value functions significantly. $\Psi = 0.96$ provides a reasonable computational time. The results below show that this approximation is not a major restriction because models with large Ψ have key counterfactual predictions.

sentation $\log(e_t^i) = \overline{e}^i + \gamma \log(e_{t-1}^i) + \sigma(1-\gamma^2)^{1/2}\epsilon_t^i$, where $\epsilon_t \sim Normal(0,1)$, i refers to a particular household, σ is the coefficient of variation, γ is the correlation coefficient, and $\overline{e}^i$ captures permanent differences in relative labor endowments and labor income. Heaton and Lucas estimate this equation using a longitudinal panel of data from the PSID that includes 860 families with data spanning the 1969-84 period. The average values of γ and σ across households are 0.53 and 0.35 respectively.

Aiyagari approximates this process using a seven state Markov Process. The approximation takes the state space of $\log(e_t)$, excluding permanent differences, to be the finite set $\{-3\sigma, -2\sigma, -\sigma, 0, \sigma, 2\sigma, 3\sigma\}$ so that the state space for e_t^i spans from $\exp(-3\sigma)$ to $\exp(3\sigma)$. Since endowments are log-normally distributed, this interval includes around 99.9% of the population. The transition probabilities are then computed by numerical integration.

4.1.1. Findings

Aiyagari studied the version $(\Gamma, \Psi) = (0, 0)$ of our model[11]. He shows that this SIM model can replicate certain features of the data but falls short of producing enough inequality. For example, the Gini coefficient of wealth is 0.32 in the model but around 0.78 in the data.

Regarding other market structures we find the following remarkable result: perfect risk sharing and perfect equality is an equilibrium when $\Gamma = 1$ for any $\Psi \in [0, 1]$. This result can be proven using Proposition 2. In Aiyagari's economy the modified golden rule level of capital-output ratio, K^*/Y^*, equals 0.79[12] while the coefficient $\frac{\beta\overline{\theta}}{1+\overline{\theta}}$ equals 0.63. Therefore, according to Proposition 2 perfect equality is a stationary equilibrium in the DC version of Aiyagari's economy. Since the Proposition assumes $\Psi = 0$, the most stringent debt limit, the result also holds for the less stringent limits.

An interpretation of this result is that debt-constraints explain *none* of the inequality in

[11] He implicitly uses $\Psi = 0$ for the quantative exercise but allows $\Psi = 1$ for the theoretical results.

[12] Remember that Y^* includes undepreciated capital. The corresponding ratio excluding undepreciated capital is 2.96.

Aiyagari's economy. All inequality in this economy is rooted in the lack of contingent assets. To better understand this finding, consider some features of the complete markets version of Aiyagari's economy reported in Table 1.a. Labor income, we_i, ranges from 0.39 to 3.14. Consumption can be equalized at the modified golden rule level if all individuals hold K^* units of riskless bonds, individuals with the lowest income realization own insurance claims of 1.87 units, and individuals with the highest income realization have insurance liabilities of 3.45 units. Agents can commit to repay this contingent debt because $(1+r^*)K^* = 5.67 > 3.45$. Thus, K^* equally distributed across the population is more than enough to secure all contingent debts required to perfectly smooth consumption.

Perfect equality would still be possible even if income risk is substantially higher. We find that perfect equality is still an equilibrium if σ equals 0.5 or if γ equal 0.75. These parameters would entail that the ratio between the highest and lowest labor income is twice the ratio in the baseline, or that log-income is 40% more persistence than the baseline. In conclusion, Aiyagari's calibration suggests that debt limits would be irrelevant if asset markets were complete. To further check the robustness of these results, we now consider two other substantially different calibrations.

4.2. Calibration 2: KP

The second calibration is a version of KP (2002). They calibrate the earnings process following the results of Storesletten, Telmer, and Yaron (2002) who argue that the persistence of earnings is around 0.97 rather than 0.53, as estimated by Heaton and Lucas. Their argument is based on the observation that the cross-sectional variance of earnings increases almost linearly with age, suggesting an almost unit root persistence of earnings. The following choice of parameters produces an earning process similar to the one considered by Krueger and Perri: $\gamma = 0.96$ and $\sigma = 0.58$. Furthermore, to be consistent with the US evidence, β is set so that the capital/output ratio, net of depreciation, is close to 3 in the

SIM model with $\Psi = 0$[13]. We call KP all economies using this calibration.

4.2.1. Findings

Table 1.b shows features of the complete markets economy under this parametrization. In contrast to Aiyagari's parametrization, perfect risk sharing and perfect equality is no longer an equilibrium. While a representative individual would like to short-sell 29.9 units payable in the event of receiving the largest labor endowment next period, she can only credibly promise to pay $(1 + r^*) K^* = 3.30$ units.

Table 2 reports features of the distributions of wealth and consumption for the US and four models under the KP calibration. The data for the US is obtained from CDR (2003), and Rodríguez et al. (2002). It is important to highlight that the data on the distribution of consumption is less reliable than the data on the wealth distribution (see Attanasio *et al.* 2004 for a discussion).

The following are some key observations regarding the distribution of wealth: *(i) all* models produce substantially more inequality than in Aiyagari's calibration. For example, Gini coefficients are significantly larger and in some cases consistent with the US evidence; *(ii)* all models nevertheless fall short of reproducing the degree of inequality in the US using measures other than the Gini coefficient. For example, while the wealthiest 1% hold around 30% of the wealth in the data, they only hold less than 10% in the model. Similarly, the coefficient of variation of wealth is 6.53 in the data but less than 1.8 in the models; *(iii)* DC models perform particularly poorly relative to SIM models. For example, DC models produce roughly half the standard deviation of wealth of SIM models (8.72 vs. 4.76 for $\Psi = 0$ and 9.69 vs. 5.28 for $\Psi = 1$); moreover, SIM models produce significant larger wealth concentration in the 5th quintile or the 99 percentile; *(iv)* Models predict more inequality as Ψ increases. For example, in the SIM model the Gini coefficient, and wealth concentration

[13]Results are similar if β is instead calibrated for each economy so that the capital/output ratio is 3.

in the 99 percentile is higher for $\Psi = 1$ than $\Psi = 0$. However, this better performance comes from a counterfactual prediction; *(v)* Models with large Ψ counterfactually predict that a significant fraction of the population has large negative wealth. For example, the share of wealth of the first quintile in the SIM model with $\Psi = 1$ is -9.19% but close to zero (-0.39%) in the data. Thus, models with a large amount of unsecured debt can better account for the top tail of the wealth distribution but at the cost of missing the bottom tail.

Regarding the distribution of consumption, we observe that: *(i)* consumption inequality is substantially less pronounced than wealth inequality in the data. For example, the Gini coefficient is 0.3; *(ii)* all models provide a reasonable fit for consumption inequality. For instance, consumption concentration in the 99 percentile is 3.77% in the data while the models predict between 2.56% to 4.33%; *(iii)* DC models tend to underpredict consumption inequality while SIM models tend to overpredict it; *(iv)* DC models perform slightly better in accounting for consumption inequality in the US.

These observations support the following conclusions: *(i)* SIM models with a zero debt limit considerably outperform the alternatives in terms of accounting for wealth inequality, and produce a reasonable fit for consumption inequality; (ii) it is harder to account for wealth inequality than for consumption inequality. Thus, matching wealth inequality is a more demanding test; (iii) the fact that the fraction of population with negative wealth is close to zero in the data cast doubt on the empirical relevance of models with unsecured debt or ample debt limits.

The next calibration accounts more closely, by construction, for the observed degree of wealth inequality.

4.3. Calibration 3: CDR

The last calibration considered is a simple version of CDR (2003). They calibrate the earning process to match the U.S. Lorenz curves of earnings and wealth. Their model allows for different realistic features of the U.S. economy regarding taxation, social security, aging, and retirement. The following parameter values are taken directly from their study: $\beta = 0.924$, $\mu = 1.5$, $\alpha = 0.376$, $\delta = 0.059$, $e = \begin{bmatrix} 0.31 & 0.98 & 3.03 & 329 \end{bmatrix}$, and

$$\pi = \begin{bmatrix} 0.9843 & 0.0117 & 0.0040 & 0.0001 \\ 0.0314 & 0.9648 & 0.0038 & 0.0000 \\ 0.0153 & 0.0044 & 0.9800 & 0.0002 \\ 0.1090 & 0.0050 & 0.0625 & 0.8235 \end{bmatrix}.$$

The parameters of the earning process correspond to that of the working population in their paper. A notable feature of this calibration is the large dispersion of labor endowments. For example, the ratio of the largest to the lowest labor endowments is $1,061$ while it is only 8 in Aiyagari's calibration. This large ratio allows a better approximation of the long tail of earnings that characterizes the U.S. Lorenz curve.

4.3.1. Findings

Table 1.c shows features of the complete markets economy under this parametrization. As in the KP calibration, perfect equality is not an equilibrium. A representative agent would need to short-sell $1,548$ units payable in the event of receiving the highest labor endowment, but she can only credibly promise to pay $(1 + r^*)K^* = 5.19$ units.

Table 3 is analogous to Table 2 but for the CDR calibration. By construction, the SIM model with $\Psi = 0$ closely approximates the distribution of wealth. Two novel results documented in Table 3 are: *(i)* DC models do not significantly improve their ability to account for the wealth distribution, even under this arguably extreme calibration of the earning process. For example, Gini coefficients, concentration in the top tail, and the coefficient of variation are similar to those reported in Table 2; *(ii)* all models now produce too much consumption inequality, particularly the SIM models.

This alternative calibration reinforces the conclusion that SIM models substantially outperform DC models in accounting for wealth inequality. However, two conclusions need to be qualified. First, relaxed debt limits remain a problem for DC models, but not for SIM models, as they counterfactually predict that too many individuals have large negative wealth. Second, SIM models are now less successful in accounting for the distribution of consumption.

4.4. The contributions of market frictions to inequality

The results above suggests that inequality arises due to both debt limits and incomplete asset markets. But what is the quantitative role of each friction in accounting for the overall inequality?

To address this question consider the SIM model with $\Psi = 0$ in Table 3 which is designed to better match the U.S. wealth inequality. One can assess the role of a particular friction by removing the friction and computing the new equilibrium level of inequality. This is precisely what is done Table 3. Comparing rows 1 and 2, one finds that removing the debt limit (keeping $\Gamma = 0$ but setting $\Psi = 1$) barely changes the predictions of the model. This supports the conclusion that the main friction explaining the benchmark level of inequality is the asset market incompleteness rather than the debt limit. Comparing rows 1 and 3 one finds that wealth inequality falls significantly when asset markets are completed (keeping $\Psi = 0$ but setting $\Gamma = 1$). Wealth concentration in the 99 percentile falls from almost 26% to around 7%, and the standard deviation of wealth falls from 29 to around 9. The results of this experiment further support the conclusion that asset market incompleteness is the main determinant of inequality.

An alternative way to assess the role of each friction is to calculate welfare measures. The welfare cost of a particular set of frictions can be defined as the value of γ_L that solves the equation

$$U\left(c^{*}\right)/\left(1-\beta\right)=\int U\left[(1+\gamma_{L})c(x,e)\right]dJ^{*}(x,e)/\left(1-\beta\right)$$

where c^{*} is the golden rule level of consumption. The left-hand side of this expression is the welfare level under perfect equality. The right-hand side of the expression is the average welfare of individuals in a distorted economy where all consumptions are adjusted by the rate γ_L. γ_L provides a lower bound of the welfare costs of frictions because all our distorted economies exhibit some consumption dispersion, which reduces welfare, but also higher average consumption, which increases welfare. γ_L can be solved as $\gamma_L=\left[\frac{U(c^{*})}{EU}\right]^{\frac{1}{1-\mu}}-1$.

A welfare measure that controls for differences in mean consumption is defined by the value of γ_H that solves

$$U\left(c^{*}\right)/\left(1-\beta\right)=\int U\left[(1+\gamma_{H})\widetilde{c}(x,e)\right]dJ^{*}(x,e)/\left(1-\beta\right)$$

where $\widetilde{c}(x,e)=c(x,e)\frac{c^{*}}{Ec(x,e)}$. This formulation satisfies $E\widetilde{c}(x,e)=c^{*}$ and $\gamma_H=(1+\gamma_L)\frac{Ec(x,e)}{c^{*}}-1$. γ_H provides an upper bound for the welfare costs of inequality.

Table 4 reports the welfare costs measures and other statistics for four models. A first observation is that the welfare costs of the frictions are quite sizable compared, say, with the welfare costs of business cycles (Lucas, 1988)[14]. Second, a consistent observation across different models is that completing the asset markets significantly improves welfare. For example, in the CDR calibration γ_L goes from 0.62 in the economy with $(\Gamma,\Psi)=(0,0)$ to 0.36 in the economy with $(\Gamma,\Psi)=(1,0)$. Third, relaxing debt-constraints may increase welfare slightly, as is the case in the CDR economy (comparing the first and second row) or actually decrease social welfare, as in the case in the KP economy. These results support the conclusion that completing the assets market may increase social welfare significantly. The

[14] Lucas is interested in the costs of fluctuation in aggregate consumption while we are interested in the costs of inequality. The main difference between the two exercises is the magnitude of the standard deviations in the execises. See Córdoba and Verdier (2007) for more welfare calculations.

same is not true if debt limits are relaxed.

5. Concluding comments

Kehoe and Levine (2001) favors the use of debt-constrained models (DC) over standard incomplete markets (SIM) models because they are simpler and produce similar results. For example, the interest rate is below the rate of time preference and there is capital overaccumulation in both models. Levine and Zame (2002) argue that market incompleteness does not matter if agents are sufficiently patient and there is no aggregate uncertainty, and Krueger and Perri (2002) argue that DC models can better explain the inequality in the U.S. In contrast with this literature, we show that DC models cannot explain key facts of the US *wealth* inequality such as the extreme wealth concentration and wealth dispersion. Moreover, DC models that rely heavily on unsecured debt also fail to explain why most debts are secured and why most households have positive net wealths.

We also document that models that rely only on asset market incompleteness can better explain wealth dispersion and wealth concentration but fail to account for the fact that most households have positive wealth. We conclude that both debt-constraints and asset market incompleteness are key components of models of inequality. However, our exercises suggest that asset market incompleteness is the main friction. It accounts better for the concentration and dispersion of wealth, and it is the most costly friction in terms of welfare.

Our results suggest that the most promising direction for future research in the area of inequality is to better understand the private information frictions that give rise to the limited supply of contingent contracts. A major challenge is to account simultaneously for the distributions of wealth and consumption. Existing models that do well in matching the wealth distribution predict too much consumption inequality.

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Table 1

Complete Market Economies $(\mathbf{\Gamma}, \mathbf{\Psi}) = (\mathbf{1}, \mathbf{1})$

(a) Aiyagari economy

$(1+r^*)k^* = 5.67$

e_i	0.32	0.46	0.66	0.93	1.33	1.88	2.66
we_i	0.39	0.55	0.78	1.10	1.56	2.21	3.14
$\theta(e_i)$	-1.59	-1.20	-0.70	-0.09	0.68	1.66	2.93
$a(e_i)$	1.87	1.40	0.82	0.11	-0.80	-1.96	-3.45

(b) KP economy

$(1+r^*)K^* = 3.31$

e_i	0.15	0.27	0.49	0.88	1.57	2.80	5.00
we_i	0.15	0.26	0.46	0.84	1.49	2.66	4.76
$\theta(e_i)$	-8.21	-6.77	-4.36	-0.44	5.94	16.21	31.40
$a(e_i)$	7.82	6.45	4.15	0.42	-5.65	-15.43	-29.9

(c) CDR economy

$(1+r^*)K^* = 5.19$

e_i	0.31	0.98	3.03	329.2
we_i	0.35	1.10	3.41	370.6
$\theta(e_i)$	-5.9	-1	23.4	1,376
$a(e_i)$	6.7	1.2	-26.3	-1,548

Table 2
KP Parametrization

The distribution of wealth

Economy	Stdev	Coef. Variat.	Gini	Quintiles 1st	2nd	3rd	4th	5th	Top Groups 90-95	95-99	99-100
US		6.53	0.78	-0.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
1. $(\Gamma,\Psi)=(0,0)$	8.72	1.51	0.66	0.02	2.34	9.09	21.72	66.83	16.55	20.57	8.27
2. $(\Gamma,\Psi)=(0,1)$	9.69	1.78	0.85	-9.19	-1.22	8.65	24.78	76.97	19.12	23.62	9.46
3. $(\Gamma,\Psi)=(1,0)$	4.76	1.17	0.59	0.45	3.13	12.11	25.19	59.12	15.03	16.78	5.96
4. $(\Gamma,\Psi^*)=(1,1)$	5.28	1.41	0.76	-8.10	0.10	12.18	28.84	66.99	16.96	18.90	6.28
The distribution of consumption											
US			0.30	7.19	12.96	17.80	23.77	38.28	9.43	9.69	3.77
1. $(\Gamma,\Psi)=(0,0)$	1.18	0.75	0.38	5.82	10.77	15.45	23.81	44.14	11.17	11.89	4.21
2. $(\Gamma,\Psi)=(0,1)$	1.22	0.78	0.40	5.00	10.43	15.51	23.89	45.17	11.38	12.21	4.33
3. $(\Gamma,\Psi)=(1,0)$	0.74	0.53	0.28	8.87	12.98	18.01	23.04	37.70	9.06	9.75	2.68
4. $(\Gamma,\Psi^*)=(1,1)$	0.71	0.51	0.28	8.84	13.56	17.17	24.00	36.43	9.40	9.50	2.56

Ψ^* refers to the debt limit $\Psi\phi^0=-we_{min}/r$.

Table 3
CDR Parametrization

The distribution of wealth

Economy	Stdev	Coeff. Variat.	Gini	Quintiles 1st	2nd	3rd	4th	5th	Top Groups 90-95	95-99	99-100
US		6.53	0.78	-0.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
1. $(\Gamma,\Psi)=(0,0)$	29.01	4.04	0.75	0.51	2.16	3.24	15.15	78.95	16.22	13.28	25.91
2. $(\Gamma,\Psi)=(0,1)$	29.22	4.10	0.79	-1.03	1.90	2.45	14.69	81.99	17.24	14.32	26.65
3. $(\Gamma,\Psi)=(1,0)$	8.82	1.55	0.66	0.75	2.92	6.06	19.68	70.58	19.15	18.75	6.77
4. $(\Gamma,\Psi^*)=(1,1)$	8.25	1.54	0.75	-13.07	5.19	18.56	21.43	67.89	18.16	17.00	6.24
The distribution of consumption											
US			0.30	7.19	12.96	17.80	23.77	38.28	9.43	9.69	3.77
1. $(\Gamma,\Psi)=(0,0)$	3.52	2.14	0.51	6.39	6.40	10.07	17.35	59.79	14.93	11.94	11.46
2. $(\Gamma,\Psi)=(0,1)$	3.47	2.07	0.49	7.19	7.22	10.06	16.88	58.64	14.76	11.89	11.24
3. $(\Gamma,\Psi)=(1,0)$	3.44	2.17	0.38	6.69	9.95	13.71	24.54	45.10	10.41	8.33	6.29
4. $(\Gamma,\Psi^*)=(1,1)$	3.24	2.10	0.33	9.89	10.16	13.24	25.31	41.40	9.47	7.57	6.01

Ψ^* refers to the debt limit $\Psi\phi^0 = -we_{min}/r$.

Table 4
Welfare Costs of Market Frictions and Inequality

	KP Calibration					CDR Calibration				
	E[k]/k*	E[c]/c*	γ_L	γ_H	$(\gamma_L+\gamma_H)/2$	E[k]/k*	E[c]/c*	γ_L	γ_H	$(\gamma_L+\gamma_H)/2$
$\Gamma=0$, $\Psi=0$	1.92	1.27	0.74	1.21	0.97	1.50	1.08	0.62	0.76	0.69
$\Gamma=0$, $\Psi=0.96$	1.80	1.26	2.37	3.24	2.81	1.48	1.10	0.54	0.70	0.62
$\Gamma=1$, $\Psi=0$	1.35	1.12	0.30	0.46	0.38	1.18	1.04	0.36	0.42	0.39
$\Gamma=1$, $\Psi=0.96$	1.24	1.11	0.28	0.43	0.36	1.12	1.01	0.28	0.30	0.29

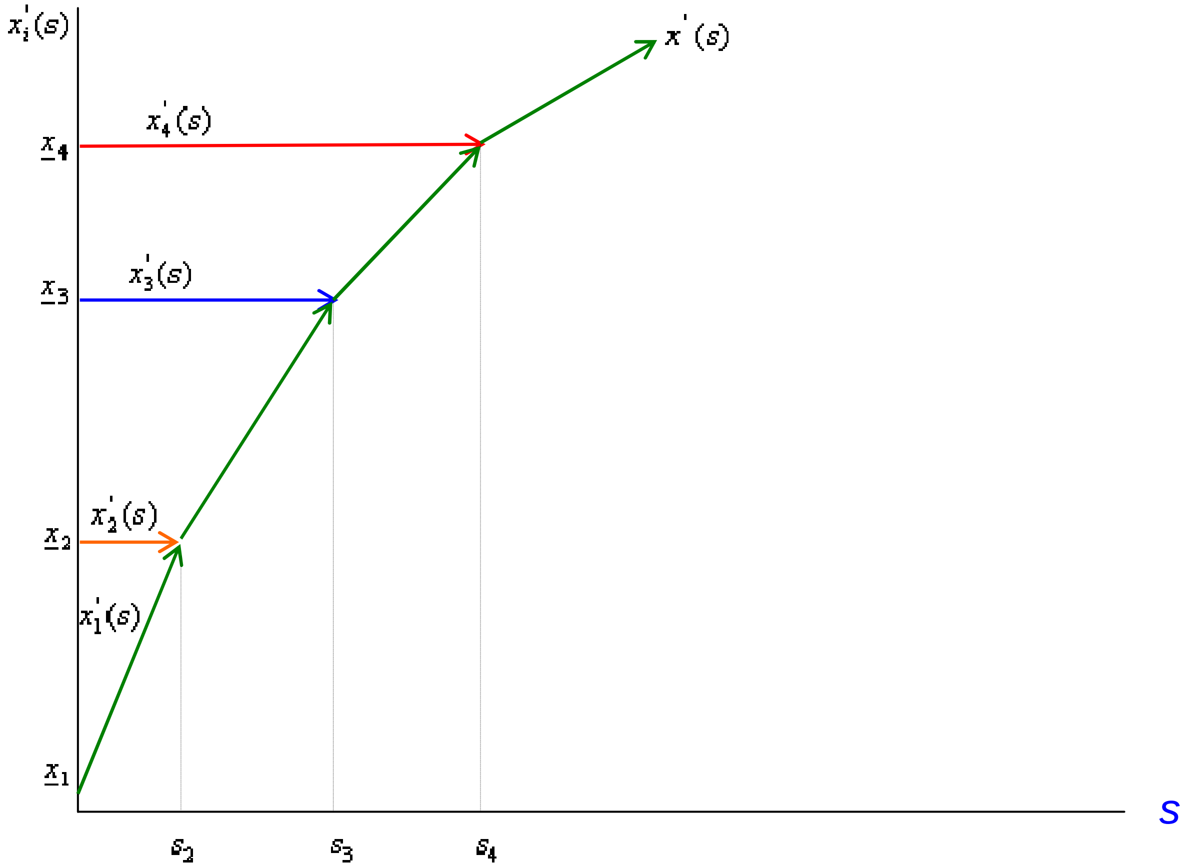

Figure 1: Allocation of savings s, i.i.d case

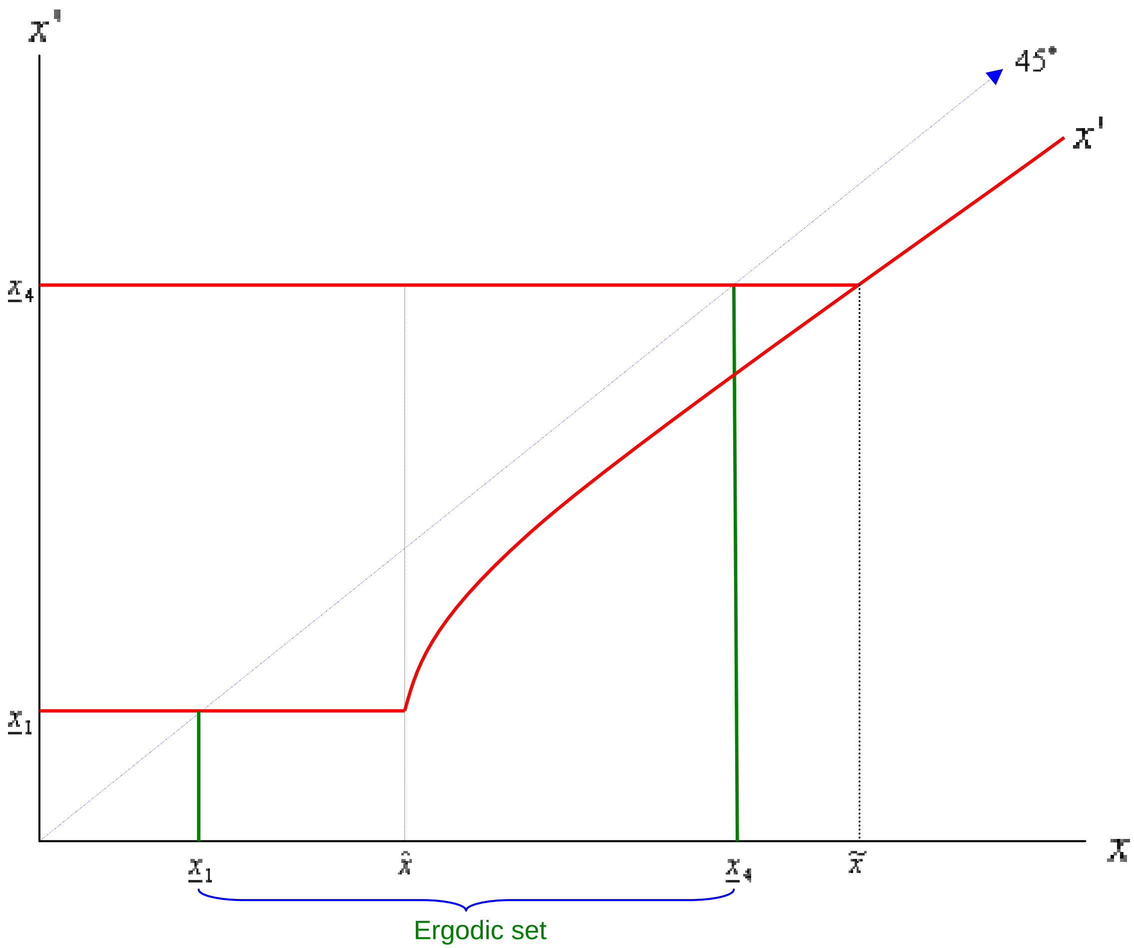

Figure 2: Evolution of resources *x* , i.i.d case

U.S. Inequality: Debt Constraints or Incomplete Asset Markets?

(Technical Appendix.)

Juan-Carlos Cordoba[a*]

[a] Rice University

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*Corresponding author: jcordoba@rice.edu

Appendix 1: Proofs

The following is a corollary of Proposition 1.

Corollary A.1 $x_{t+1} \geq \underline{x}_n$ requires $s \geq s_n$.

A household optimal total savings, s, is characterized by the Euler equation:

$$u_c(c) \geq \beta(1+r)Eu_c(c') \tag{1}$$

The following two propositions characterize the ergodic set of x.

Proposition A.2 Suppose $r < \rho$. Then, (i) if $x_t \geq \underline{x}_n$, then $x_{t+1} < x_t$; and (ii) if $x_t \leq \underline{x}_n$ then $x_{t+1} \leq \underline{x}_n$.

Proof. (i) Suppose, by contradiction, that $x_{t+1} \geq x_t \geq \underline{x}_n$. By Corollary A.1, $s_t = g(x_t) \geq s_n$. Proposition 1 then implies that $x'^{*}_{jt+1} = s_t(1+r)+\omega$ for all j. Thus, $x_{t+1} = s_t(1+r)+\omega$ and c_{t+1}, a sole function of x_{t+1}, is equal in all possible states. For unconstrained agents, condition (1) then reads $u_c(c_t) = \beta(1+r)u_c(c_{t+1})$. Since $r < \rho$, then $c_{t+1} < c_t$, which requires $x_{t+1} < x_t$, a contradiction. (ii) Suppose by contradiction that $x_{t+1} > \underline{x}_n \geq x_t$. A similar argument to part (i) implies that $x_{t+1} < x_t$, a contradiction. ■

Proposition A.3 Suppose $r < \rho$. Then, $g(\underline{x}_1) = 0$.

Proof. Suppose, by contradiction that $g(\underline{x}_1) > 0$, which implies $x'_1 > \underline{x}_1$. From the first order condition with respect to x'_1, $u_c(c(\underline{x}_1)) = \beta(1+r)u_c(c(x'_1))$, with equality because $x'_1 > \underline{x}_1$. Then $u_c(c(\underline{x}_1)) < u_c(c(x'_1))$, or $c(\underline{x}_1) > c(x'_1)$, or $\underline{x}_1 > x'_1$, a contradiction ■

The following Lemma is stated without proof as the results are well known.

Lemma A.4 If debt constraints are not binding for any positive mass of agents in equilibrium, then the stationary level of capital is determined by the modified golden rule, individual consumptions are constant over time, and the distribution J is not unique. In particular, a perfectly equalitarian distribution of consumptions and riskless assets is an equilibrium.

Proof of Proposition 2.. Suppose perfect risk sharing is a stationary equilibrium. Then, using the budget constraint, $a_t(e_t)$ satisfies:

$$a_t(e_t) = c + b - we_t - (1+r)b + \sum \beta a_{t+1}(e_{t+1})\pi(e_{t+1}|e_t),$$

where consumption and holdings of riskless assets are constant, and $q(e', e) = \beta\pi(e_{t+1}|e_t)$ has been used. Iterating on this equation produces:

$$a_t(e_t) = \frac{c - rb}{1-\beta} - w\sum_{s=0}^{\infty} \beta^s E_t\left[e_{t+s}|e_t\right]$$

Without loss of generality, by Lemma A.4, consider a representative agent of this economy who holds $b = K^*$ and $c = f(K^*) - K^*$. Then, $c - rb = w$ and:

$$a_t(e_t) = w\sum_{s=0}^{\infty} \beta^s \left(1 - E_t\left[e_{t+s}|e_t\right]\right) = -w\theta(e_t)$$

Since perfect risk sharing is possible, then $a_t(e_t) > -(1+r)K^* = K^*/\beta$ for all e_t, or $-w\theta > K^*/\beta$. Using the result $w = f(K^*) - K^*/\beta$, one finds that $\frac{K^*}{f(K^*)} > \frac{\beta\bar{\theta}}{1+\bar{\theta}}$. For sufficiency, one can go backwards and construct a stationary equilibrium with a representative agent. ■

Proof of Proposition 3. From the first order conditions we obtain

$$u_c(c) \geq \beta(1+r^b)Eu_c(c').$$

If $\beta(1+r^b) > 1$ then $K^b < K^*$ and $M_t \geq E_t M_{t+1}$, where $M_t \equiv \beta^t(1+r^b)^t u_c(c_t)$. M_t is a non-negative supermartingale and therefore converges. Since $\beta(1+r^b) > 1$, then u_c must converge to zero. Thus, $c \to \infty$ which violates feasibility since $K^b < K^*$. If $\beta(1+r^b) = 1$ then $u_c(c) \geq E u_c(c')$. Thus, $u_c(c)$ converges. If c converges to a finite constant, then full insurance is obtained which contradicts Proposition 2. Thus, $c \to \infty$ which violates feasibility. Then $\beta(1+r^b) < 1$, which requires $K^b > K^*$. ∎

Appendix 2: Computational Algorithm

The solution of the SIM model is standard and follows Aiyagari (1994). The DC problem can be described as:

$$V(x,e) = \max_{s \geq 0} \left\{ u(x-s) + \beta \sum_{i=1}^{n} V(x_i(s,e), e_i)\pi(e_i|e) \right\}$$

where $x_i(s,e)$ is the optimal allocation rule for next period state i. This problem is solved using value function iterations, which requires to start with an initial guess about V and obtaining an updated value function using the equation above. The challenge in this problem is that in order to update V, one needs first to find $x_i(s,e)$ given the current guess about V.

We find $x_i(s,e)$ using a procedure motivated by Figure 1. For the i.i.d case, $x_i(s,e)$ is a piecewise linear function fully characterized by the nodes at which new assets start being demanded. Figure 3 illustrates a case of persistent shocks. Contrary to the i.i.d case, interior solutions are such that the resources allocated to bad states should be larger than those allocated to good states, since bad and good states are expected to persist. Thus, the increasing segments do not overlap as in the i.i.d case. Moreover, the increasing segments do not need to be strictly linear as in the i.i.d case. However, Figure 3 suggests that $x_i(s,e)$ can be approximated by piecewise linear functions of well defined nodes. Those nodes, denoted by x_j^i, can be found from the solution to the following equations:

$$V'(x_j^i, e_j) = V'(\underline{x}_i, e_i) \text{ for } i,j \in \{1,..,n\}.$$

The right-hand side of this equation is the marginal utility of the minimum level of state i resources. Thus, x_j^i is the level of state j's resources that equates state j's marginal utility to state i marginal utility (evaluated at the minimum level). Given V, the equations above can be solved numerically and the nodes can be computed. The nodes are then used to compute $x_i(s,e)$ using a modified version of Proposition 1.